Program Manual
for
OPERATIONAL MINIMUM VARIANCE TRACKING AND ORBIT PREDICTION PROGRAM

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Prepared for

SPECIAL PROJECTS BRANCH
THEORETICAL DIVISION
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

Prepared by

SPERRY RAND SYSTEMS GROUP
SPERRY GYROSCOPE COMPANY
DIVISION OF SPERRY RAND CORPORATION
GREAT NECK, NEW YORK

Sperry Report No. AB-1210-0022

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I. INTRODUCTION

This report describes an operational version of a computing program for orbit determination based on an extension of the Schmidt-Kalman minimum variance technique for processing tracking data. This operational version, by members of the Technical Staff of the Sperry Rand Systems Group, Sperry Gyroscope Company, Great Neck, N. Y. was done on contract NAS5-3509 under the direction of R. K. Squires and D. S. Woolston of the Theoretical Division, Goddard Space Flight Center.

The analysis on which the prototype program was based was carried out by Samuel Pines and Henry Wolf of Analytical Mechanics Associates, Inc. with assistance from R. K. Squires and D. S. Woolston of Goddard Space Flight Center, NASA, and from Mrs. A. Bailie of AMA. The prototype digital program was written by John Mohan of AMA. The program makes use of portions of a least squares orbit determination program written by Miss E. Fisher of Goddard Space Flight Center. Significant contributions to the program have been made by J. Behuncik, G. Wyatt, E. R. Lancaster and F. Shaffer of Goddard Space Flight Center.

This report is partly composed of material prepared by D. S. Woolston (NASA) and John Mohan (AMA) and presented in GSFC report X-640-63-144, dated July 1, 1963.

II. NOTATION

This section presents a partial list of symbols used in this report. Most symbols are defined in the section in which they are used; some of the more important which are used in several sections are presented here.

A	azimuth angle
A	area of satellite normal to velocity vector
$[A]$	precession transformation matrix
C_D	drag coefficient
DEC	declination
E	elevation angle
F_1, F_2, F_3, F_4	perturbing forces acting on vehicle
$[G]$	general purpose orthogonal transformation matrix
H	angular momentum vector
HA	hour angle
IP	integral part of
$\left.\begin{matrix} J_2, J_3, J_4 \\ J_{20}, J_{30}, J_{40} \end{matrix}\right\}$	zonal harmonic coefficients
K(t)	Kalman weighting function
$[L]$	libration matrix
L(t)	see equation 20
M(t)	matrix relating $\overrightarrow{\Delta\alpha}$ to errors in observation $\Delta\gamma$, including $M_A, M_E, M_{p'}, M_{\dot{p}}, M_h, M_d, M_l, M_m, M_x, M_y$
N(t)	matrix relation $\overrightarrow{\Delta\alpha}$ to errors in observation $= M(t)S$
$[N]$	nutation transformation matrix

N_e	electron density in ionosphere
$P(t), P(t_o)$	covariance matrix of $\vec{\Delta x}$ at present and initial times, respectively
$Q(t), Q(t_o)$	covariance matrix of NASA parameters at present and initial times, respectively
Q_o	$Q(t_o)$
R_{VC}	vector between dominant body and vehicle
R_{TB}	local two body position vector
$R_o, \dot{R}_o$	initial position and velocity vectors
R_{SE}	station location vector in system not including precession and nutation
R_B	vehicle position vector in base date system
R_{SB}	ground station position vector in base date system
RA	right ascension
S	point transformation matrix $\frac{\partial x}{d\alpha}$
V_B	velocity of vehicle in base date system
V_{VC}	velocity of vehicle with respect to reference body
V_{SB}	velocity of station in base date system
$[W_e]$	earth rotation vector
X_B, Y_B, Z_B	unit vectors in base date system, $\hat{X}$ toward Aries, $\hat{Y}$ normal to $\hat{X}$ in mean equator of base date, $\hat{Z}$ normal to $\hat{X}, \hat{Y}$
$Y(t)$	major semiaxis
a	major semiaxis
a_e	earth's equatorial radius
b, c	see equation 7
c	speed of light

d	declination
d_o	$= Ro \cdot \dot{R}o$
$d-d_{50}$	= days from epoch to 0^h Jan 1, 1950
$\hat{e}_B, \hat{h}_B, \hat{n}_B$	east, up and north unit vectors in base date system
$\hat{e}_B'', \hat{n}_B''$	east. and north vectors after correction for transmission time
$\hat{e}_B''', \hat{n}_B'''$	modified east and north vectors of $R/\dot{R}$ system
f, g, f_t, g_t	functions relating position and velocity vectors at an initial time to these vectors at a later time
h_G	height of station above the geoid
h	magnitude of angular momentum vector
h	hour angle
i	orbital inclination
$\hat{k}$	unit vector in z direction
m	vehicle mass
n	mean motion
r	magnitude of R vector
r_{TB}	magnitude of R_{TB} vector
r_{vc}	magnitude of R_{VC} vector
r_o	magnitude of Ro vector
r_r	magnitude of R vector at rectification
t_o	initial time
t	present time
t_r	time of last rectification
Δt	$t - t_o$
Δt	round trip range time (Section V)

$\Delta t'$	range rate time (Section V)
t_{max}	length of time from epoch over which program is to operate
t_{1max}	time limit on first iteration
t_{2max}	time limit on subsequent iterations
t_d	time of a data point
$\left.\begin{matrix}\Delta u\\ \Delta v\\ \Delta w\end{matrix}\right\}$	corrections to station locations in u,v,w system
v_o	magnitude of initial velocity vector
$\vec{\Delta x}$	variation in X
α	rotation angle between true north and arbitrary north of R/$\dot{R}$ system
α	β^2/a
$\Delta\alpha$	variation in NASA parameters
β	$\sqrt{\lvert a\rvert}\ \theta$
$[\gamma]$	general rotation matrix
γ	Greenwich hour angle
γ_M	difference between predicted and computed observations
$\bar{\epsilon}^2$	covariance matrix of observation instrument
θ	differential eccentric anomaly
$\Lambda(t,t_r)$	approximate parameter transition matrix modified to include rectification effects
λ_G	longitude of station
μ	mass times gravitational constant

ξ	vector between two body and actual orbital position at t
ρ	atmospheric density
ρ	one way range
ρ'	two way range
$\dot{\rho}$	range rate (one way)
$\vec{\rho}_B$	station to vehicle range vector in base date system
$\vec{\rho}_B''$	station to vehicle vector after station location is corrected for transmission time
σ	standard deviation of measurements
τ_ρ	one way transmission time
$\Phi(t,t_o)$	exact state transition matrix
ϕ_G	station longitude
$\phi(t,t_o)$	approximate (two-body) state transition matrix
$\psi(t,t_o)$	approximate (two-body) parameter transition matrix
λ_G	longitude of station
Ω	earth rotation vector
$\Omega(t,t_o)$	exact parameter transition matrix

Analysis

III. ANALYTICAL BACKGROUND

A detailed discussion and derivation of the Schmidt-Kalman minimum variance technique as modified and extended for use in the present program is contained in Reference 1. In the present document emphasis will be placed on the working equations required to implement the method. Some essential concepts and distinctive features of the method should be introduced, however, and are described briefly in this section.

A. Orbit Prediction Program

For the purpose of predicting position as a function of time the program makes use of Encke's form of the differential equations of motion. It therefore solves analytically the best local two-body problem and numerically integrates the deviation from this two-body reference trajectory. When the true trajectory departs from the reference orbit by preset limits, a rectification is performed; that is, the current true radius and velocity vectors are used to define a new reference path.

B. Solution of the Two-Body Problem

In the particular solution of the two-body problem used herein problems associated with circular orbits and orbits of very low inclination have been eliminated by expressing the solution in terms of initial position and velocity vectors rather than vectors based on position of perigee.

C. Orbit Parameters

The choice of the elements used in the differential correction scheme is of utmost importance in predicting observations and other orbit functions over a long time period. It may be shown (Reference 2) that the best choice for parameters is that in which only one variable affects the energy or the mean motion of the orbit. The conventional astronomical elements have this property. However, three of these variables, the argument of perigee, the time of perigee passage, and the ascending node become poorly defined for near circular and low inclination orbits. The initial position and velocity components do not have this difficulty. However, all six of these affect the energy. A convenient set of parameters has been derived in Ref. 3. These avoid the difficulties for circular and low inclination orbits as well as restricting the energy parameters to a single element. The variational parameters are as follows:

$\Delta\alpha_1(t)$ A rigid rotation of R about $\dot{R}$ such that $R \cdot \dot{R}$ remains constant.

$\Delta\alpha_2(t)$ A rigid rotation of $\dot{R}$ about R such that $R \cdot \dot{R}$ remains constant.

$\Delta\alpha_3(t)$ A rigid rotation of R and $\dot{R}$ about the angular momentum vector, H.

$\Delta\alpha_4(t)$ A change in the variable $\frac{R \cdot \dot{R}}{\sqrt{\mu |a|}}$ such that the angle between R and $\dot{R}$ is changed leaving the magnitude of R and $\dot{R}$ and also "a" unchanged.

$\Delta\alpha_5(t)$ A change in $\frac{1}{a}$, the reciprocal of the semi-major axis, such that the magnitudes of R and $\dot{R}$ are changed, leaving the eccentricity unchanged.

$\Delta\alpha_6(t)$ A change in the variable $1 - \frac{r}{a}$, changing the magnitude of R and $\dot{R}$ and the angle between them such that "a" and $R \cdot \dot{R}$ are not changed.

These six elements have the characteristic that they determine the orbit independently of its orientation or shape and do not break down. Moreover, the matrix of partial derivatives of these elements contains only one secular term, namely that due to the semi-major axis, a.

D. The Schmidt-Kalman Technique

Since the orbit position and velocity are not directly observable, it is necessary to infer these variables from a sequence of observations which are functions of the trajectory. In the conventional methods, a linear relationship is assumed between the deviations in the observations and the corresponding deviations in the orbit variables. Thus, an error in the orbit position will correspond to some predictable error in the observation. A large number of observations are made, overdetermining the linear system of equations. A least square technique is used to obtain the best value of the orbit errors to fit the known observation errors. Since the equations of motion are essentially nonlinear, this region of linearity becomes more and more constrictive about the nominal trajectory the longer the time period over which the prediction is made. Thus, the least square technique often produces a result, fitting data over a long time arc, which is outside the linear range. This produces problems in convergence and consumes machine time. Reducing the number of observations to a shorter time arc helps avoid this difficulty. The weighted least squares is often used in this manner. However, a large number of observations is always needed in order to properly evaluate the effect of the random instrument errors.

The method of least squares and weighted least squares both relate the estimate of the initial parameters to an entire sequence of observational residuals spread over an extended time arc. In contrast, the method of minimum variance relates the present estimate of the state variable deviation to the present actual deviations in the observations. The linear assumptions required for the updating theory are violated to a much less degree in the method for minimum variance than in the method of weighted least squares.

E. Closed Form Analytical Derivatives

The requirement for utilizing closed form analytical derivatives is associated with the need for rapid computing time. If the program were required to integrate the variations in the observations due to changes in the orbit parameters directly from the differential equations for these variations, the computing time over and above that required for the nominal trajectory would increase by a factor of six (6). Since these variations are only required in order to obtain small iterative changes to the orbit parameters, approximate expressions will be useable, provided the residual in the observations can be accurately computed. This situation is analogous to the possible use of an approximate derivative in Newton's method for obtaining the roots of a polynomial. The program presented in this report uses a set of analytical derivatives based on the two-body problem approximation of the osculating orbit given in terms of the parameters outlined earlier. In a manner similar to the Encke method, a readjustment is made in the partial derivatives whenever the orbit is rectified.

IV. ORBIT PREDICTION PROGRAM

A. Equations of Motion

The equations of motion of a vehicle with negligible mass under the action of a dominant central force field and perturbed by other smaller forces is given by:

$$\ddot{R}_{VC} = -\frac{\mu}{r^3} R_{VC} + F_1 + F_2 + F_3 + F_4 \qquad 1)$$

These equations may be written in the Encke form by replacing the vector R_{VC} by the sum of a local two-body orbit position vector, R_{TB}, plus a perturbation displacement,

$$R_{VC} = R_{TB} + \xi \qquad 2)$$

The vector R_{TB} satisfies the differential equations,

$$\ddot{R}_{TB} = -\mu \frac{R_{TB}}{r_{TB}^3} \qquad 3)$$

The Encke equation of motion for the perturbation displacement, ξ, is given by:

$$\ddot{\xi} = -\mu\left(\frac{R_{VC}}{r_{VC}^3} - \frac{R_{TB}}{r_{TB}^3}\right) + F_1 + F_2 + F_3 + F_4 \qquad 4)$$

These are the equations that will be integrated to obtain a precision nominal trajectory.

The perturbations that are included in this program are those due to the gravitational attraction of the sun, spherical moon, Venus, Mars and Jupiter (F_1); the program also includes the perturbations due to the earth's oblateness (F_2) and the perturbations due to atmospheric drag (F_3). The effect of lunar oblateness is added as (F_4).

B. Computation of Perturbation Terms

The equations for the gravitational perturbation acceleration due to the sun, moon, and planets are given by:

$$F_1 = -\sum_{i=1}^{5} \mu_1 \left(\frac{R_i}{r_i^3} - \frac{R_{ci}}{r_{ci}^3} \right) \tag{5}$$

The perturbation accelerations due to the earth's oblateness are given by:

$$F_2 = b\, R_{VC} + c\hat{K} \tag{6}$$

where k is a unit vector in the z-direction and where

$$b = \left[\frac{\mu J_2}{r^5}\left(-1 + 5\left(\frac{z}{r}\right)^2\right) + \frac{\mu J_3}{r^6}\,\frac{5}{2}\left(3\frac{z}{r} - 7\frac{z^3}{r^3}\right) + \frac{15\mu J_4}{r^7}\left(-1 + 14\frac{z^2}{r^2} - 21\frac{z^4}{r^4}\right)\right] \tag{7}$$

$$C = \mu z\left[-\frac{2J_2}{r^5} + \frac{J_3}{r^5 z}\left(-\frac{3}{2} + \frac{15}{2}\frac{z^2}{r^2}\right) + \frac{20 J_4}{r^7}\left(-3 + 7\frac{z^2}{r^2}\right)\right]$$

in which r is the magnitude of the vector R_{VC}.

The perturbations due to atmospheric drag are given by

$$F_3 = \frac{1}{2}\rho \frac{AC_D}{m}\left(\dot{R}_{VC} - \Omega \times R_{VC}\right)\left|\dot{R}_{VC} - \Omega \times R_{VC}\right| \tag{8}$$

The vector $\dot{R}_{VC} - \Omega \times R_{VC}$ is the velocity of the vehicle relative to the atmosphere rotating rigidly with the earth. The vector Ω is the earth rotation vector and contains only a component in the z direction. Its magnitude is given by the earth's siderial rotation rate.

The perturbation due to lunar oblateness, and the effects of lunar libration and earth's nutation and precession on F_2 are given in Appendix A.

C. <u>Computation of the Encke Term</u>

A special problem arises in the computation of the Encke term due to the loss of accuracy in subtracting the nearly equal terms involved. An expression based on the binomial expansion removes this difficulty. The method supplies results more accurate than the straightforward computation for terms of the type

$$\frac{R}{r^3} - \frac{R_0}{r_0^3}$$

if $|R - R_0|$ is small compared to r and is known more accurately than can be computed by taking the difference between R and R_0.

One can write

$$\frac{R}{r^3} - \frac{R_0}{r_0^3} = \frac{R}{r^3} - \frac{R}{r_0^3} + \frac{R}{r_0^3} - \frac{R_0}{r_0^3}$$

$$= \frac{R}{r^3}\left[1 - \left(\frac{r}{r_0}\right)^3\right] + \frac{\Delta R}{r_0^3}$$

$$= \frac{\Delta R}{r_0^3} + \frac{R}{r_3}\left[1 - (1+\mu)^{3/2}\right]$$

or finally:

$$\frac{R}{r^3} - \frac{R_0}{r^3} = \frac{\Delta R}{r_0^3} + \frac{R}{r^3}\sum_{n=1}^{6} a_n u^n \qquad 9)$$

where

$$u = \frac{2}{r_0^2}\left(R_0 + \frac{1}{2}\Delta R\right)\cdot \Delta R$$

$$a_1 = -\frac{3}{2}, \quad a_2 = -\frac{3}{8}, \quad a_3 = \frac{1}{16}, \qquad 10)$$

$$a_4 = -\frac{3}{128}, \quad a_5 = \frac{3}{256}, \quad a_6 = -\frac{7}{1024}$$

The six terms are adequate for $|u| < 0.1$. For larger values of u straightforward computation is adequate.

D. Solution of the Two-Body Problem

The vector position and velocity for a Kepler orbit may be written in terms of the initial position and velocity vectors as given in Reference 4.

$$R = f R_o + g \dot{R}_o$$
$$\dot{R} = f_t R_o + g_t \dot{R}_o \qquad 11)$$

Thus, the expressions for f, g, f_t and g_t, applicable to hyperbolic, parabolic, and elliptic trajectories provide the basis for a flexible trajectory computation system.

Herrick's variables provide the means for achieving this goal. The technique, due to Herrick (Ref 17), is as follows:

Let initial conditions $X_o = (R_o, \dot{R}_o)$ be given for some time t_o and suppose it is required to find the value of $X = (R, \dot{R})$ on the same two-body trajectory at some time $t = t_o + \Delta t$. The quantities

$r_o = |R_o|$, $v_o = |\dot{R}_o|$, $d_o = R_o \cdot \dot{R}_o$, and $\frac{1}{a} = \frac{2}{r_o} - \frac{v_o^2}{\mu}$ are first computed. The next step is to transform the elapsed time, Δt, into a more easily handled variable denoted by either θ or β. This is done by solving Kepler's equation implicitly for the new variable. The forms taken by Kepler's equation for elliptic, hyperbolic, and parabolic trajectories are as follows:

$$\sqrt{\mu}\,\Delta t = a^{3/2}(\theta - \sin\theta) + r_o\sqrt{a}\,\sin\theta + \frac{d_o}{\sqrt{\mu}}a(1-\cos\theta)$$
$$\sqrt{\mu}\,\Delta t = (-a)^{3/2}(\sinh\theta - \theta) + r_o\sqrt{-a}\,\sinh\theta + \frac{d_o}{\sqrt{\mu}}(-a)(\cosh\theta - 1) \qquad 12)$$
$$\sqrt{\mu}\,\Delta t = \frac{1}{6}\beta^3 + r_o\beta + \frac{1}{2}\frac{d_o}{\sqrt{\mu}}\beta^2$$

For the elliptic and hyperbolic cases, define

$$\beta = \sqrt{|a|} \cdot \theta$$

$$\alpha = \beta^2 \left(\frac{1}{a}\right)$$

$$f_1(\alpha) = \frac{1}{6} - \frac{\alpha}{120} + \frac{\alpha^2}{5040} - \dots = \sum_0^\infty \frac{(-\alpha)^i}{(2i+3)!}$$

$$f_2(\alpha) = \frac{1}{2} - \frac{\alpha}{24} + \frac{\alpha^2}{720} - \dots = \sum_0^\infty \frac{(-\alpha)^i}{(2i+2)!}$$

$$f_3(\alpha) = 1 - \alpha \cdot f_1$$

$$f_4(\alpha) = 1 - \alpha \cdot f_2$$

Kepler's equation assumes for all three types of trajectories the form $\sqrt{\mu}\,\Delta t = \beta^3 f_1 + r_0 \beta f_3 + \frac{d_0}{\sqrt{\mu}} \beta^2 f_2$

Once β, or θ, is known, functions f, g, f_t, g_t are computed. In the elliptic, hyperbolic, and parabolic cases these are defined by

<u>elliptic</u>

$$f = 1 - \frac{a}{r_0}(1 - \cos\theta)$$

$$g = r_0 \sqrt{\frac{a}{\mu}} \sin\theta + \frac{a d_0}{\mu}(1 - \cos\theta) \qquad 13)$$

$$f_t = -\frac{\sqrt{\mu a}}{r\, r_0} \sin\theta$$

$$g_t = 1 - \frac{a}{r}(1 - \cos\theta)$$

where

$$r = a(1 - \cos\theta) + r_0 \cos\theta + \frac{d_0}{\sqrt{\mu}} \sqrt{a} \sin\theta;$$

hyperbolic

$$f = 1 - \frac{|a|}{r_0}(\cosh\theta - 1)$$

$$g = r_0\sqrt{\frac{|a|}{\mu}}\sinh\theta + \frac{|a|d_0}{\mu}(\cosh\theta - 1)$$

$$f_t = \frac{-\sqrt{\mu|a|}}{r r_0}\sinh\theta \qquad 14)$$

$$g_t = 1 - \frac{|a|}{r}(\cosh\theta - 1)$$

where $r = |a|(\cosh\theta - 1) + r_0\cosh\theta + \frac{d_0}{\sqrt{\mu}}\sqrt{|a|}\sinh\theta$; and

parabolic

$$f = 1 - \frac{1}{2r_0}\beta^2$$

$$g = \frac{r_0}{\sqrt{\mu}}\beta + \frac{d_0}{2\mu}\beta^2$$

$$f_t = -\frac{\sqrt{\mu}}{r r_0}\beta \qquad 15)$$

$$g_t = 1 - \frac{1}{2r}\beta^2$$

where $r = \frac{1}{2}\beta^2 + r_0 + \frac{d_0}{\sqrt{\mu}}\beta$.

The general form, good for all three types of trajectories is

$$f = 1 - \frac{1}{r_0}\beta^2 f_2$$

$$g = \frac{r_0}{\sqrt{\mu}}\beta f_3 + \frac{d_0}{\mu}\beta^2 f_2$$

$$f_t = -\frac{\sqrt{\mu}}{r r_0}\beta f_3$$

$$g_t = 1 - \frac{1}{r}\beta^2 f_2$$

where $$r = \beta^2 f_2 + r_0 f_4 + \frac{d_0}{\sqrt{\mu}} \beta f_3$$

Thus, having f, g, f_t, and g_t, X is obtained from the equations

$$\begin{aligned} R &= f R_0 + g \dot{R}_0 \\ \dot{R} &= f_t R_0 + g_t \dot{R}_0 \text{ .} \end{aligned} \qquad 11)$$

Also,

$$r = |R| = \beta^2 f_2 + r_0 f_4 + \frac{d_0}{\sqrt{\mu}} \beta f_3$$

and

$$\frac{d}{\sqrt{\mu}} = \frac{R \cdot \dot{R}}{\sqrt{\mu}} = \left(1 - r_0 \left(\frac{1}{a}\right)\right)\beta \; f_3 + \frac{d_0}{\sqrt{\mu}} f_4$$

E. Integration and Rectification Control

The Encke method reduces somewhat the relative advantages of one integration scheme over another insofar as numerical accuracy is concerned. The method is capable of using almost any integration scheme to obtain a precise solution. The major advantage to be gained in the choice of integration schemes lies in the choice of the maximum integration interval to minimize the total computing time required. The Encke method computes the solution of the equations of motion as a sum of the exact function plus the integrated effect of the perturbations. Thus, the solution may be kept as precise as the exact portion so long as the accumulated error in the integrated portion is kept from affecting the least significant digit of the exact term. By estimating the accumulated round-off error and the accumulated truncation error in the integrated portion of the solution, and by rectifying the solution to a new osculating Kepler orbit whenever the integrated error threatens to affect the least significant digit of the exact solution, the total solution may be kept as precise as the exact term can be computed.

The particular program outlined in this report uses a fourth order Runge-Kutta integration scheme to initialize a sixth order backward difference second sum Cowell integration formula. A constant step size is used in place of a variable integration interval. At pre-set points in the trajectory the optimum interval size is altered, based on previous numerical experience with these intervals.

The rectification feature outlined above, based on round-off error control, is presently not in the program. At present, rectification is triggered whenever the integrated portion of the solution is a fixed ratio of the exact two-body term. In effect, this controls the accumulation of round-off error.

V. THE MODIFIED SCHMIDT-KALMAN METHOD

A. The Statistical Filter

A modification of the Schmidt-Kalman equations in terms of the new orbit parameters selected for use in the program has been derived in References 1 and 3 and is available for incorporation in the orbit determination program.

The deviations of the orbit variables in terms of the new parameters are given by

$$\Delta \mu(t) = \frac{\partial \mu}{\partial \alpha} \Delta \alpha(t) = S(t) \Delta \alpha(t)$$

where S(t) is a point transformation matrix.

The parameter transition matrix Ω (t,t_0) is defined by

$$\Delta \alpha(t) = \frac{\partial \alpha(t)}{\partial \alpha(t_0)} \Delta \alpha(t_0) = \Omega(t,t_0) \Delta \alpha(t_0) \qquad 16)$$

The observation errors in terms of the new parameters are given by

$$\Delta \gamma(t) = N(t) \ \Delta \alpha(t) \qquad 17)$$

where

$$N(t) = M(t) \ S(t)$$

The corresponding covariance matrices are given by

$$\begin{aligned} E(\Delta\alpha, \Delta\alpha^*) = Q(t) &= \Omega(t,t_0)\, Q(t_0)\, \Omega^*(t,t_0) \\ P(t) &= S(t) \ Q(t) \ S^*(t) \\ Y(t) &= N(t) \ Q(t) \ N^*(t) + \overline{\varepsilon}^2 \end{aligned} \qquad 18)$$

The inverse relationship between the orbit parameter corrections and the observations errors is given by

$$\Delta\alpha(t) = L(t)\,\Delta\gamma(t) \tag{19}$$

The optimum filter L(t) is given by

$$L(t) = Q(t)\,N^{*}(t)\,Y^{-1}(t) \tag{20}$$

The corrected covariance matrix after each observation is given by

$$Q(t^{+}) = Q(t^{-}) - Q(t^{-})N^{*}(t^{-})\,Y^{-1}(t^{-})\,N(t^{-})\,Q(t^{-}) \tag{21}$$

Using these equations, it is now possible to use the Schmidt-Kalman scheme for both short and long term predictions.

At the start of the program, the input matrix, Po, is given in terms of the state vectors. Since the program computes in variational parameters, an initial conversion is required, i.e.,

$$Q(t_o) = S^{-1}(t_o)\;P(t_o)\;S^{-1*}(t_o) \tag{22}$$

Since S^{-1} is an analytic quantity, no machine inversion is needed.

The only other inversion is the Y matrix which never exceeds dimensions of 4 x 4; this inversion does not constitute a computational burden.

B. The Point Transformation Matrix

The inverse point transformation matrix, $S^{-1} = \frac{\partial \alpha}{\partial X}$, is given in reference 1 as

$$S^{-1} = \begin{bmatrix} -\frac{V}{h^2} H & 0 \\ 0 & \frac{r}{h^2} H \\ 0 & \frac{H \times \dot{R}}{h v^2} \\ \frac{H \times R}{a^2 n r^2} - \frac{a}{r^3} \alpha_4 \alpha_6 R & -\frac{H \times \dot{R}}{a^2 n v^2} - \frac{2a}{r v^2} \alpha_4 \alpha_6 \dot{R} \\ \frac{-2R}{r^3} & \frac{-2\dot{R}}{\mu} \\ \frac{V^2}{\mu r} R & \frac{2r}{\mu} \dot{R} \end{bmatrix} \qquad 23)$$

Each element in this matrix represents a row vector having three components. By choosing $\frac{1}{a}$ as a parameter, the other five parameters will automatically be independent of the energy providing the inverse of the matrix $\left(\frac{\partial \alpha}{\partial 4}\right)$ exists. This is guaranteed by defining the transformation matrix S(t) such that

$$(S^{-1}) S = I \qquad 24)$$

The point transformation matrix S(t) is given by

$$\Delta \psi = \begin{bmatrix} \dfrac{\partial \psi}{\partial \alpha} \\ \\ \dfrac{\partial \dot{\psi}}{\partial \alpha} \end{bmatrix} \Delta \alpha = S \Delta \alpha$$

$$S = \begin{bmatrix} -\frac{H}{V} & 0 & \frac{H \times R}{h} & \frac{a^2 n}{h^2} H \times R & -aR & -\frac{a}{r} R + \frac{\mu^2 \alpha_6 \alpha_4 |a| H \times R}{h^2 r^2 V^2 n a} \\ \\ 0 & \frac{H}{r} & \frac{H \times \dot{R}}{h} & 0 & \frac{a}{2} \dot{R} & \frac{\mu a}{r^2 V^2} \dot{R} \end{bmatrix} \qquad 25)$$

Each element in this matrix represents a column vector with three components.

C. <u>State Transition Matrix</u>

The method of obtaining the state transition matrix is based on generalization of an Encke method applied to linear prediction theory. It is assumed that the equations of motion may be decomposed into two factors

$$\dot{x} = g(x,t) + h(x,t)$$

where

$$g \gg h$$

It is further assumed that a closed form solution of the differential equations is known for the case where h = 0,

$$\dot{s} = g(s,t)$$

Furthermore, the state transition matrix for the approximating solution is known in closed form

$$\Delta s(t) = \phi(t,t_0)\, \Delta s(t_0) \tag{26}$$

Let the deviation between the state variable and its approximation be given by

$$p(t) = x(t) - s(t)$$

The perturbation equations of motion may now be written in the generalized Encke form

$$\dot{p} = g(x,t) - g(s,t) + h(x,t)$$

In order to guarantee that the deviation, p, is never permitted to grow too large, the process of rectification is introduced. Whenever a predetermined value of p is exceeded, the integration is terminated at time t_r. A new set of initial conditions are introduced, setting $p(t_r)$ equal to zero. Integration proceeds again about this new nominal approximate solution.

Since the deviation between x and s is never permitted to exceed the given value, the partial derivatives of the state variables from their nominal value may also be limited. Thus it is possible to write an approximate state transition matrix

$$\phi(t,t_0) \cong \Phi(t,t_0) \tag{27}$$

D. <u>The Parameter Transition Matrix</u>

If the exact or the approximate state transition matrix were available in the program, the parameter transition matrix, which is $\frac{\partial \alpha(t)}{\partial \alpha(t_0)}$, could be computed from

$$\begin{aligned} \frac{\partial \alpha(t)}{\partial \alpha(t_0)} &= \frac{\partial \alpha(t)}{\partial \mu(t)} \frac{\partial \mu(t)}{\partial \mu(t_0)} \frac{\partial \mu(t_0)}{\partial \alpha(t_0)} \\ &= S^{-1}(t)\, \Phi(t,t_0)\, S(t) \\ &\cong S^{-1}(t)\, \phi(t,t_0)\, S(t_0) \end{aligned} \tag{28}$$

Thus, if Ω (t,t_0) is the exact parameter transition matrix and ψ (t,t_0) is the approximate one, then

$$\begin{aligned} \Omega(t,t_0) &= S^{-1}(t)\, \Phi(t,t_0)\, S(t_0) \\ \psi(t,t_0) &= S^{-1}(t)\, \phi(t,t_0) \end{aligned} \tag{29}$$

where S and S^{-1} are evaluated along the exact trajectory in computing Ω, and along the approximating conic when computing ψ

However, it is desirable to obtain ψ directly instead of from the state transition matrix; consequently the latter matrix never appears in the program. Derivation of the parameter transition matrix is discussed in Ref. 8. The resulting matrix is given by the following equation

$$\begin{bmatrix} \frac{f\,V}{V_r} & -g\frac{V}{r_r} & 0 & 0 & 0 & 0 \\ -f_t\frac{r}{V_r} & g_t\frac{r}{r_r} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ell_{34} & \frac{3}{2}\frac{\mu h a}{r^3 V^2}(t-t_r) & \ell_{36} \\ 0 & 0 & 0 & g_t & \frac{3}{2}\frac{\mu}{nar}\alpha_6(t)(t-t_r) & -\frac{r_r}{an}f_t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{ang}{r} & \frac{3}{2}\frac{a^2 n}{r}\alpha_4(t)(t-t_r) & \frac{r_r f}{r} \end{bmatrix} \qquad 30)$$

The terms ℓ_{34} and ℓ_{36} are:

$$\ell_{34} = \frac{h}{V^2}a^2 n\left[\frac{g_t-1}{r^2} - \frac{r_r f_t^2}{\mu} - \frac{f_t}{h^2}\left(\frac{\mu}{r}g - d_r\right)\right] \qquad 31)$$

$$\ell_{36} = -\frac{h}{V^2}\frac{a}{r_r}f_t\left[\frac{r_r}{r}f_4(\theta) + \frac{r_r^2}{r^2} - \frac{\mu g_t}{r_r V_r^2} + \frac{\mu d_r}{h^2 r_r V_r^2}\alpha_6(t_r)\left(\frac{\mu}{r}g - d_r\right)\right]$$

To obtain the parameter transition matrix over a time interval greater than one rectification interval, the following equation applies

$$\psi(t,t_o) = \psi(t,t_r)\, S_{ar}^{-1}(t_r) \quad S_{br}(t_r) \quad \psi(t_r, t_o) \qquad 32)$$

where the (br) and (ar) subscripts mean "before rectification" and "after rectification", respectively.

E. <u>Iteration in the Modified Schmidt-Kalman Method</u>

The basic concept of the method provides a recursive technique for establishing the most up to date estimate of the vehicle's position and velocity. After an initial transient condition has settled out (as data are accrued) the estimate is essentially independent of the initial state vector ($X_o \rightarrow \dot{Z}_o$) and covariance matrix (Q_o) employed. In cases where a post flight analysis of the best estimate of the trajectory from start to finish is needed, the initial transitory conditions need resolution. This is done by an iterative technique to be described herein.

A time limit, t_{1max}, is introduced. In the first pass through the data from t_o to t_{1max}, the initial covariance matrix Q_o is transformed to Q_{tmax} and the initial state vector $\vec{X}_o$ is transformed to $\vec{X}_{tmax}$. The Q matrix at t_{max} is translated back to its equivalent at t_o. Data are not processed in this backward translation. The equation

$$Q_o = \psi(t_o, t_{max}) \; Q_{tmax} \; \psi^*(t_o, t_{max}) \qquad 33)$$

serves as the necessary translation.

At the same time, the state vector $\vec{X}_{tmax}$ is translated back to $\vec{X}_{to}$ by backward integration from t_{max} to t_o using $\vec{X}_{tmax}$ as the starting condition.

The data are then reprocessed, using this state vector, and either the new Q_o matrix or the original Q_o matrix to derive a better estimate of the trajectory. This iterative loop can be repeated as many times as the user desires. After the preselected number of iterations, an option is available to set t_{1max} ahead to t_{2max}, thereafter repeating the whole procedure to get a more refined orbit from additional data.

F. Matrix Flow in the Modified Schmidt-Kalman Method

The matrix flow in this program has a number of variations depending upon whether rectifications with or without data are involved. A step by step description of the flow from t_o to t_{max} is given here, assuming that rectifications with and without data are encountered in the process. The procedure is also followed from t_{max} back to t_o.

The input covariance matrix is P_o and the outputs are also P matrices. Since the internal computations are all in terms of the Q matrix, conversions are used in input/output.

Step 1: Convert P_o to Q_o

$$Q_o = S^{-1}(t_o)\, P_o\, S^{-1*}(t_o) \qquad 34)$$

Step 2: Integrate and find that an Encke rectification is required at t_r

Compute $\psi(t_r, t_o)$

Compute $S_{br}(t_r)$ - subscript br is "before rectification"

Rectify

Compute $S_{ar}^{-1}(t_r)$ - subscript ar is "after rectification"

Let $$\Lambda(t_r, t_o) = S_{ar}^{-1}(t_r)\, S_{br}(t_r)\, \psi(t_r, t_o) \qquad 35)$$

Step 3: Integrate from t_r to data point t_d

Compute $\psi(t_d, t_r)$

Compute $$\psi(t_d, t_o) = \psi(t_d, t_r)\Lambda(t_r, t_o) \qquad 36)$$

Step 4: Translate Q_o to $Q(t_d^-)$

$$Q(t_d^-) = \psi(t_d, t_o)\, Q_o\, \psi^*(t_d, t_o) \qquad 37)$$

Step 5: Translate Q across a data point

$$Q(t_d^+) = Q(t_d^-) - Q(t_d^-)N^*(t_d)\left[N(t_d^-)Q(t_d^-)N^*(t_d^-) + \epsilon^2\right]^{-1} N(t_d^-)\,Q(t_d^-) \qquad 38)$$

Step 6: For print-out purposes, compute $P(t_d^+)$

$$P(t_d^+) = S_{br}(t_d)\, Q(t_d^+)\, S_{br}^*(t_d) \qquad 39)$$

Step 7: Integrate to subsequent point, either data, non-data rectification, or t_{max}

Rectify after data are assimilated in Step 5

Set ψ matrix to I

Integrate to next point (called t' here to indicate any one of the three conditions)

Compute $\psi(t', t_d)$

Compute $S_{br}(t')$

Rectify at t'

Compute $S_{ar}^{-1}(t')$

Let $\Lambda(t', t_d) = S_{ar}^{-1}(t') \, S_{br}(t') \, \psi(t', t_d)$ 40)

Step 8: Translate $Q(t_d)$ to t'

$$\bar{Q}(t') = \Lambda(t', t_d) \, Q^{+}(t_d) \, \Lambda^{*}(t', t_d) \quad 41)$$

Step 9: Translate $Q(t_{max})$ back to t_o, assuming that an Encke rectification occurs

Backward integrate until rectification is indicated

Compute $\psi(t_r, t_{max})$

Compute $S_{br}(t_r)$

Rectify

Compute $S_{ar}^{-1}(t_r)$

Let $\Lambda(t_r, t_{max}) = S_{ar}^{-1}(t_r) \, S_{br}(t_r) \, \psi(t_r, t_{max})$

Continue integrating to t_o

Compute $\psi(t_o, t_r)$

Compute $S_{br}(t_o)$

Rectify

Compute $S_{ar}^{-1}(t_o)$

Let $\Lambda(t_o, t_r) = S_{ar}^{-1}(t_o)\, S_{br}(t_o)\, \Psi(t_o, t_r)$ 43)

$$\Psi(t_o, t_{max}) = \Lambda(t_o, t_r)\, \Lambda(t_r, t_{max}) \quad 44)$$

$$Q(t_o) = \Psi(t_o, t_{max})\, Q(t_{max})\, \Psi^*(t_o, t_{max}) \quad 45)$$

Step 10: For print-out purposes, compute $P(t_o)$

$$P(t_o) = S_{br}(t_o)\, Q(t_o)\, S^*_{br}(t_o) \quad 46)$$

G. <u>Computation of Observables</u>

1. Observation Types and Definitions

The program will accept the following types of observational data, singly or in combination.

1. Azimuth
2. Elevation
3. Range
4. Range Rate
5. Hour Angle
6. Declination
7. l direction cosine
8. m direction cosine
9. X angle
10. Y angle
11.* Δ t (round trip range equivalent)
12.* Δ t'(one way range rate equivalent)
13. open
14. open
15. open
16. open

Definition of measurements are:

<u>Azimuth, A,</u> is pictured in Figure I, in which vehicle position, station position and station orientation are shown at the nominal <u>observation time</u>. The observation time is defined to be the time

*Not operational at this writing.

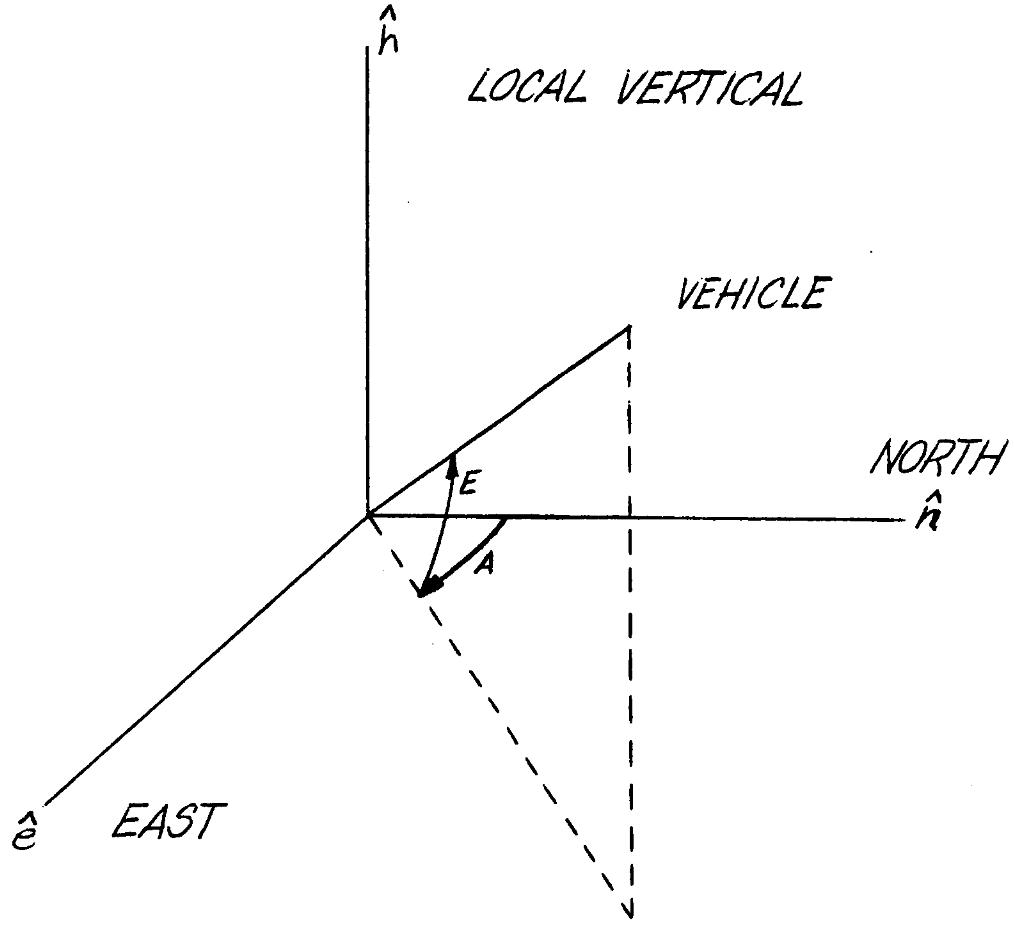


FIGURE I: AZIMUTH AND ELEVATION

the radar signal leaves the vehicle. Calculated azimuth is measured from north in the true horizontal plane positive eastward from 0 to 2π (geodetic azimuth).

Elevation, E, is also defined in Figure I. The observation time, as for Azimuth, is defined to be the time the signal leaves the vehicle. Calculated elevation is measured from the horizontal plane, positive upward, within the range $\pm \pi/2$ (geodetic elevation).

ρ', Round-Trip Range, is twice the straight line distance from station to vehicle at the nominal time of observation, t_o. The units calculated are ER.

Range Rate, $\dot{\rho}$, is the time derivative of the magnitude of the vector from station to vehicle at the nominal instant of observation. The calculated range rate is in units of ER/HR.

Hour Angle, HA, is the angle between the station meridian and the projection on the true equator of the station-to-vehicle vector measured in the earth's true equatorial plane. The convention for the calculation of HA is that it is measured from station meridian positive westward from 0 to 2π. Figure II shows the angles HA and DEC.

Declination, DEC, is the angle made with the true equatorial plane by the station-to-vehicle vector. Declination is measured positive in the northern hemisphere, with limits $\pm \pi/2$.

ℓ-Direction Cosine, is the cosine of the angle between the station-to-vehicle vector and the topocentric system east-vector,

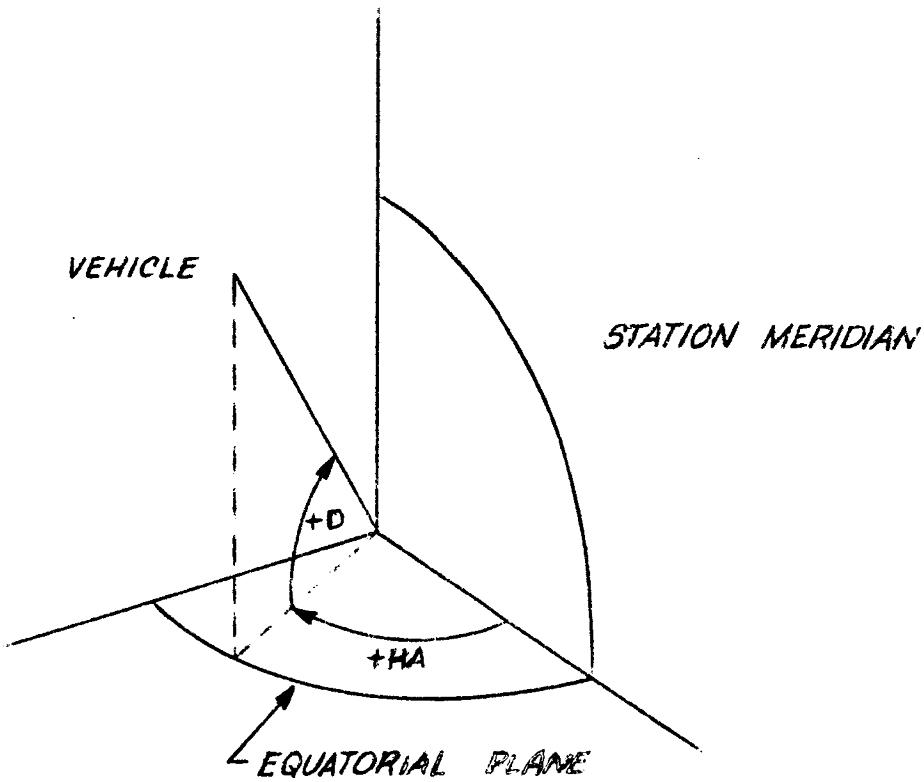


FIGURE II: HOUR ANGLE AND DECLINATION

at the nominal observation time. The topocentric system east vector is taken to lie in the local horizontal plane, normal to the local meridian, positive eastward (in the southern as well as the northern hemisphere). It has limits of ±1.0 in the computation, with no dimensions. Figure III shows l and m.

m-Direction Cosine, is the cosine of the angle between the station-to-vehicle vector and the topocentric system north vector, at the nominal observation time. The north vector is taken to lie in the local horizontal plane and in the meridian plane, positive in the north direction (in the southern as well as the northern hemisphere). It has limits of ±1.0, with no dimensions.

Antenna Pointing Angle, Y, shown in Figure IV, is the angle between the station-to-vehicle vector and the perpendicular projection of this vector on the plane formed by the $(\pi/2+\alpha)$ and vertical vectors.

Antenna Pointing Angle, X, shown in Figure IV, is the angle between the vertical vector and the perpendicular projection of the station-to-vehicle vector on the plane formed by the $(\pi/2+\alpha)$ and vertical vectors.

2. Observation Limits and Units

The limits and units given in the above definitions apply only to the calculated observations while internal to the machine. When printed out the units and dimensions are as shown in Table I. This table also gives the limits and units currently assumed for

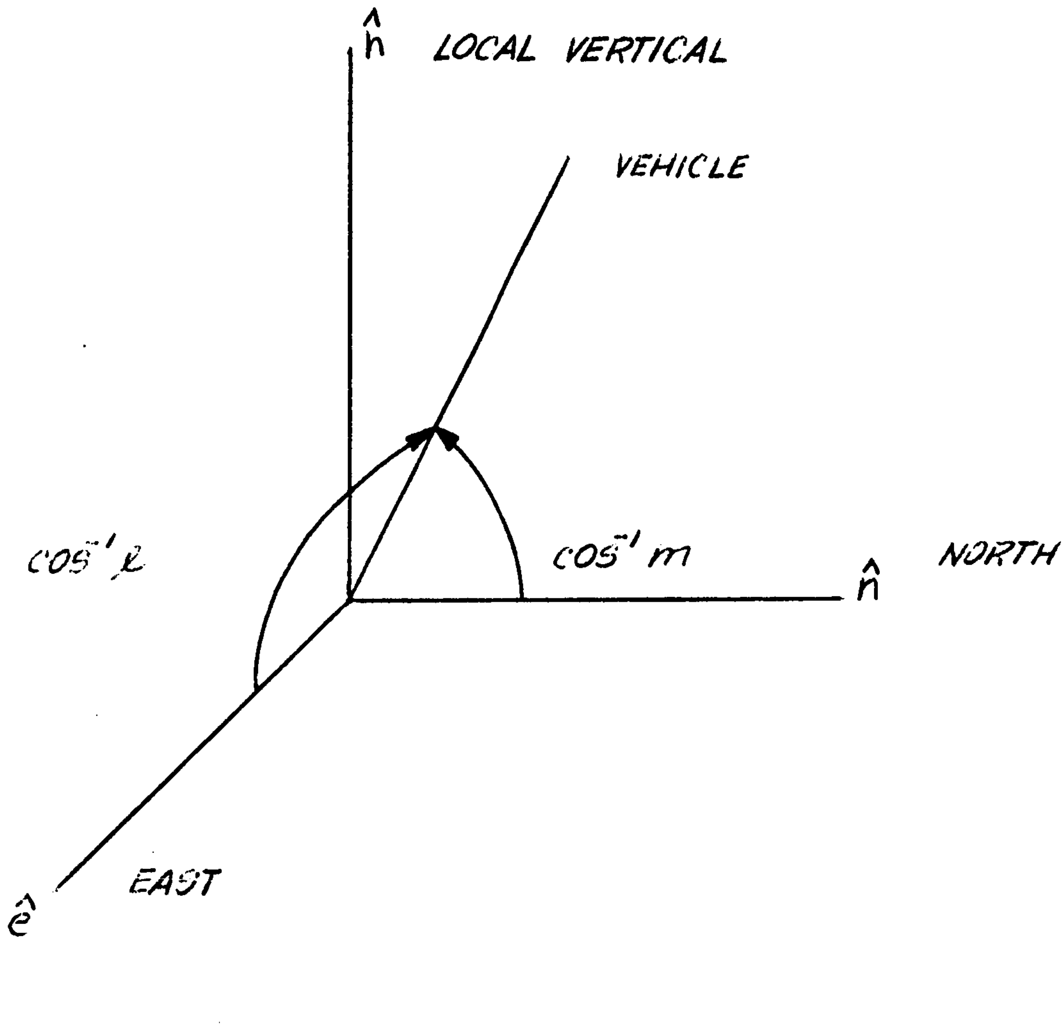


FIGURE III: DIRECTION COSINES ℓ AND m

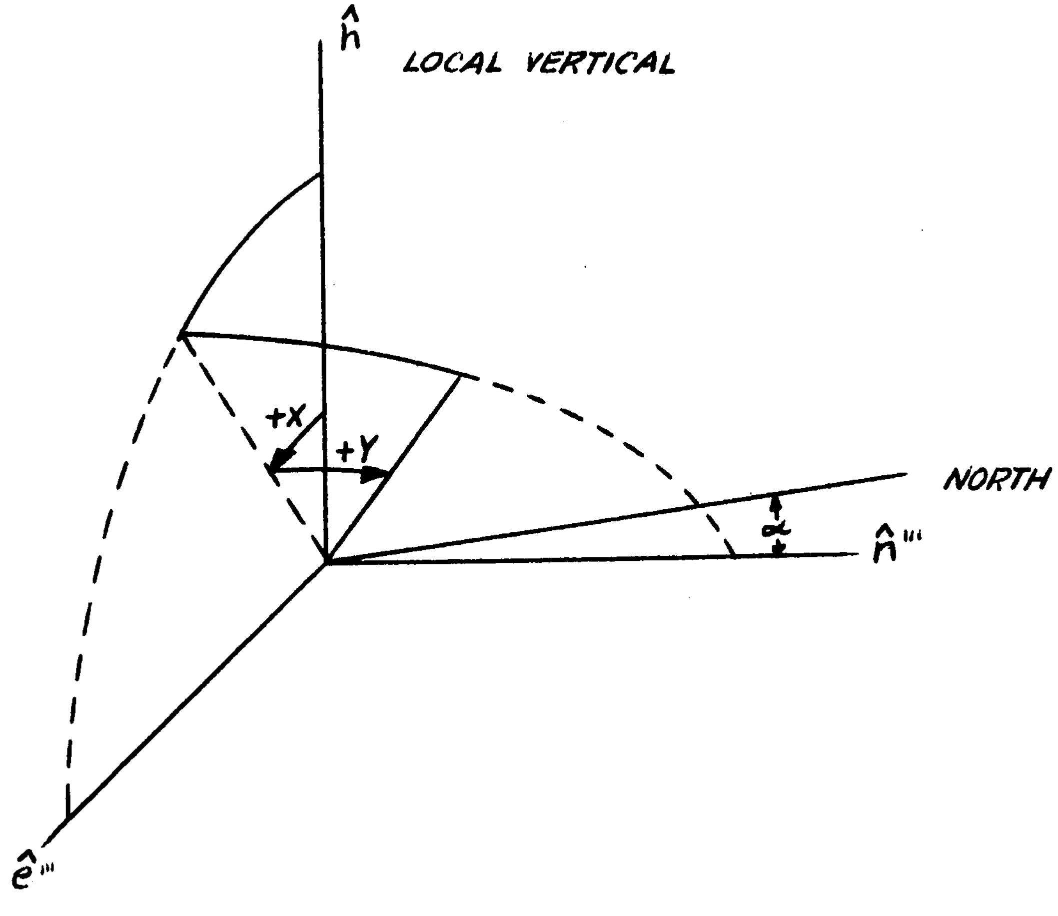


FIGURE IV: ANTENNA X AND Y

TABLE I

LIMITS AND UNITS FOR OBSERVATIONS

OBSERVATION TYPE	Calculated		Data		
	Internal	Print-Out	Input	Internal	Print-Out
Azimuth	$0,\ 2\pi$ rad	$0^\circ,\ 360^\circ$	$0^\circ, 360^\circ$	$0, 2\pi$ rad	$0^\circ, 360^\circ$
Elevation	$0, \pm \frac{\pi}{2}$ rad	$0^\circ,\ 90^\circ$	$0^\circ,\ 90^\circ$	$0, + \frac{\pi}{2}$ rad	$0^\circ,\ 90^\circ$
Round Trip Range	ER	ER or KM*	SEC	ER	ER and KM
Range Rate	ER/HR	ER/HR or KM/SEC*	SEC	ER/HR	ER/HR or KM/SEC
Hour Angle	$0,\ 2\pi$ rad	$0^\circ,\ 360^\circ$	$0^\circ, 360^\circ$	$0, 2\pi$ rad	$0^\circ, 360^\circ$
Declination	$0, \pm \frac{\pi}{2}$ rad	$0^\circ,\ 360^\circ$	$0^\circ, 360^\circ$	$0, \pm \frac{\pi}{2}$ rad	$0^\circ, 360^\circ$
l cosine	±1.0	±1.0	±1.0	±1.0	±1.0
m cosine	±1.0	±1.0	±1.0	±1.0	±1.0
Antenna x	$0, \pm \pi$ rad	$0^\circ,\ \pm 180^\circ$	$0^\circ,\ \pm 90^\circ$	$0, \pm \frac{\pi}{2}$ rad	$0^\circ,\ \pm 90^\circ$
Antenna y	$0, \pm \frac{\pi}{2}$ rad	$0^\circ,\ \pm 90^\circ$	$0^\circ,\ \pm 90^\circ$	$0, \pm \frac{\pi}{2}$ rad	$0^\circ,\ \pm 90^\circ$

Notes: KM: kilometers
ER: earth radii
HR: hours
SEC: seconds

*KM/SEC is obtained by specifying KLM = 1 in INPUT. Otherwise KLM = 0 must be specified, giving ER and ER/HR at the output.

real data observations. The observation data are converted to the internal units by the INPUT subroutine

3. Nutation, Precession and Time Correction

In order to generate the corrections in the Kalman Filter, it is necessary to compute the difference between the measured data and a computed estimate of the measured quantity based on previous information. Since these data types are referenced to earth-based systems and the vehicle's position is referenced to inertial space, nutation and precession corrections are required to compute the true inertial position of the ground station.

The time assigned to each type of data is, by definition, the time at which the data signal is transmitted from the satellite. By contrast, the time recorded on the data messages is the time at which the data signal is received at the earth's surface. Consequently, in order to compute residuals, it is necessary to obtain the assigned time from the recorded time. This calculation is different for different data types.

For true range measurement, such as obtained from conventional pulsed radar, the time correction is obtained from $\Delta t = \rho/c$ where ρ is the one-way range and c is the velocity of light. The same correction may be used with NASA's range and range-rate system provided the range ambiguities are resolved before making the time correction.

Ground station location in inertial space is specified by the time the signal is received at the station. For the highly accurate MINITRACK data, the ℓ and m direction cosine residuals are computed using corrected assigned time for satellite position and recorded time for station location. For less accurate angular measurements, such as h, d, X, Y, ground station location is computed at the assigned time. In systems measuring round-trip range and range-rate, the average station position from transmission to reception corresponds to the position at the assigned time, and this average inertial location is used in computing the residuals.

4. Formulas for Calculation of Observations

Taking into account precession and nutation, the observables are computed with the following equations. Definition of applicable matrices are given in Section VI-3 and in Appendix A. Correction for the effect of refraction must be added to the quantity listed here to get the best estimate of the observable. The corrections are discussed in Appendix C.

The station location vector in the coordinate system without nutation and precession is

$$\vec{R}_{SE} = [\gamma] \left([G]_S \begin{bmatrix} 0 \\ 0 \\ h_G + C \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \epsilon^2 C \sin \phi_G \end{bmatrix} + \begin{bmatrix} \Delta \mu \\ \Delta v \\ \Delta w \end{bmatrix} \right) \qquad 47)$$

The station location vector, including precession and nutation, is

$\overrightarrow{R_{SB}} = [A]\,[N]\,\overrightarrow{R_{SE}}$	station in $X_B\ Y_B\ Z_B$	48)
$\overrightarrow{\rho_B} = \overrightarrow{R_B} - \overrightarrow{R_{SB}}$	station to vehicle position vector	49)
$\Delta u,\ \Delta v,\ \Delta w$	geodetic net correction	
$\rho = \lvert\overrightarrow{\rho_B}\rvert$	station to vehicle distance	50)
$\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	east vector, in station coordinates	51)
$\hat{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	north vector, in station coordinates	52)
$\hat{z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	up vector, in station coordinates	53)
$\hat{e}_B = [A]\,[N]\,[\delta]\,[G]_s\,\hat{x}$	east vector, in base date system	54)
$\hat{n}_B = [A]\,[N]\,[\delta]\,[G]_s\,\hat{y}$	north vector, in base date system	55)
$\hat{h}_B = [A]\,[N]\,[\delta]\,[G]_s\,\hat{z}$	up vector, in base date system	56)

(1) Azimuth

$$A = \operatorname{Tan}^{-1}\left[\frac{\overrightarrow{\rho_B}\cdot\hat{e}_B}{\overrightarrow{\rho_B}\cdot\hat{n}_B}\right] \qquad 0 \leq A \leq 2\pi \qquad 57)$$

the quadrant of A is obtained from the signs of numerator and denominator

(2) Elevation, E

$$E = TAN^{-1} \frac{\vec{\rho}_B \cdot \hat{h}_B}{\left[(\vec{\rho}_B \cdot \hat{c}_B)^2 + (\vec{\rho}_B \cdot \hat{n}_B)^2\right]^{1/2}} \qquad -\frac{\pi}{2} < E < \frac{\pi}{2} \tag{58}$$

the quadrant of E is obtained from the signs of numerator and denominator

(3) Round Trip Range, ρ'

$$\rho' = 2\rho \tag{59}$$

(4) Range Rate, $\dot{\rho}$

$$\vec{V}_{SB} = [A][N][W_e]\ \vec{R}_{SE} \tag{60}$$

velocity of station in base date system

$$\vec{V}_B' = \vec{V}_B - \vec{V}_{SB} \tag{61}$$

velocity of vehicle relative to station in base date system

$$\dot{\rho} = \vec{V}_B' \cdot \vec{\rho}_B / \rho \tag{62}$$

range rate

(5) Hour Angle, h

$$RA = TAN^{-1}\left[\frac{\vec{\rho}_B \cdot \hat{Y}}{\vec{\rho}_B \cdot \hat{X}}\right] \tag{63}$$

Right Ascension of station-to-vehicle vector

$0 \leq RA \leq 2\pi$, quadrant obtained from signs of numerator and denominator

$$h = \Upsilon + (\lambda_G)_S - RA \tag{64}$$

Hour Angle of station-to-vehicle vector

$0 \leq h < 2\pi$

(6) Declination, d

$$d = Tan^{-1}\left[\frac{\vec{\rho}_B \cdot \hat{Z}_B}{\left[(\vec{\rho}_B \cdot \hat{X}_B)^2 + (\vec{\rho}_B \cdot \hat{Y}_B)^2\right]^{1/2}}\right] \qquad -\pi/2 \leq d \leq \pi/2 \tag{65}$$

(7) Minitrack, ℓ

$$\ell = \frac{\vec{\rho}_B'' \cdot \hat{e}_B''}{\rho_B''}$$

$$\tau_\rho = \rho / c$$

$$\vec{\rho}_B'' = \vec{\rho}_B - \vec{V}_{SB}\tau_\rho \qquad 66)$$

$$\hat{e}_B'' = \hat{e}_B + [W_e]\hat{e}_B \tau_\rho$$

$$\hat{n}_B'' = \hat{n}_B + [W_e]\hat{n}_B \tau_\rho$$

$$\rho_B'' = |\vec{\rho}_B''|$$

c = speed of propagation

(8) Minitrack, m

$$m = \frac{\vec{\rho}_B'' \cdot \hat{n}_B''}{\rho_B''} \qquad 67)$$

(9) Range Antenna, X

$$X = \tan^{-1} \frac{\vec{\rho}_B \cdot \hat{e}_B'''}{\vec{\rho}_B \cdot \hat{h}_B} \qquad -\pi \le X \le \pi \qquad 68)$$

quadrant obtained from signs of numerator and denominator

$$\hat{e}_B''' = [A][N][\theta][G]_s[\alpha] \quad \hat{x}$$

$$\hat{n}_B''' = [A][N][\theta][G]_s[\alpha] \quad \hat{y}$$

$$[\alpha] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(10) Range Antenna, Y

$$Y = \tan^{-1}\left[\frac{\vec{\rho}_B \cdot \hat{n}_B'''}{\left[(\vec{\rho}_B \cdot \hat{e}_B''')^2 + (\vec{\rho}_B \cdot \hat{h}_B)^2\right]^{1/2}}\right] \qquad -\frac{\pi}{2} \le Y \le \frac{\pi}{2} \qquad 69)$$

(11) Δ t (round trip range equivalent)

$$\Delta t = \rho' / c \qquad 70)$$

(12) Δ t' (one way range rate equivalent)

$$\Delta t' = \frac{\rho_2 - \rho_1}{c}$$

where ρ_2 is the range evaluated at the end of the measurement; ρ_1 is the range evaluated at the start of the measurement

5. Formulae for Calculation of Partial Derivatives

In addition to determining the residuals, partial derivatives relating the variation of the observables to the variation of the orbit parameters are required.

The most convenient way for computing these partials is to compute them with respect to the state variables, $X \rightarrow \dot{Z}$, then convert them to the variational parameters by the S matrix.

M = partial of observations with respect to the state variables

N = MS = partial of observations with respect to variational parameters 72)

The S matrix has been considered in a preceding section.

The M matrices are as follows: the first three elements of each row have the dimension of the observation x $(ER)^{-1}$, the last three have the dimension of the observation x (HR/ER).

1. Azimuth

$$M_A = \frac{1}{\rho \cos E}\left[\left(\hat{e}_B \cos A - \hat{n}_B \sin A\right), \vec{0}\right] \tag{73}$$

2. Elevation

$$M_E = \frac{1}{\rho}\left[\left(-\hat{e}_B \sin E \sin A - \hat{n}_B \sin E \cos A + \hat{h}_B \cos E\right), \vec{0}\right] \tag{74}$$

3. Round Trip Range

$$M_{\rho'} = 2\left[\frac{\vec{\rho}_B}{\rho}, \vec{0}\right] \tag{75}$$

4. One Way Range Rate

$$M_{\dot\rho} = \frac{1}{\rho}\left[\left(\vec{V}_B - \frac{\dot\rho}{\rho}\vec{\rho}_B\right), \ \vec{\rho}_B\right] \tag{76}$$

5. Hour Angle, h

$$M_h = \frac{-1}{\rho\cos d}\left[[A][N]\begin{Bmatrix}-\sin RA\\ \cos RA\\ 0\end{Bmatrix}, \ \vec{0}\right] \tag{77}$$

6. Declination, d

$$Md = \frac{1}{\rho}\left[[A][N]\begin{Bmatrix}-\cos RA\sin d\\ -\sin RA\sin d\\ \cos d\end{Bmatrix}, \ \vec{0}\right] \tag{78}$$

7. ℓ Direction Cosine

$$M_\ell = \frac{1}{\rho_B''}\left[\left(\hat{e}_B'' - \ell\,\frac{\vec{\rho}_B''}{\rho_B''}\right), \ \vec{0}\right] \tag{79}$$

8. m Direction Cosine

$$M_m = \frac{1}{\rho_B''}\left[\left(\hat{n}_B'' - m\,\frac{\vec{\rho}_B''}{\rho_B''}\right), \ \vec{0}\right] \tag{80}$$

9. X Angle

$$M_X = \frac{1}{\rho\cos Y}\left[\hat{e}_B''' \cos X - \hat{h}_B\sin X), \ \vec{0}\right] \tag{81}$$

10. Y angle

$$M_Y = \frac{1}{\rho}\left[\hat{n}_B''' \cos Y - \hat{e}_B'''\sin Y\sin X - \hat{h}_B\sin Y\cos X), \vec{0}\right] \tag{82}$$

$$\left.\begin{array}{l}\text{11. } \Delta t\\ \text{12. } \Delta t'\end{array}\right\} \text{ M matrices not presently included}$$

VI. ADDITIONAL PROGRAM FEATURES

A. Initial Condition Transformations

1. Introduction

Transformation of vehicle initial conditions and earth and lunar oblateness attractions to the "base date" coordinate system are included in MINIVAR. The "base date" system is determined by the direction of the vernal equinox of $0^h.0$ January 0 of the year subsequent to the launch year. It has been chosen as the basis for calculation because the planetary and solar coordinates are written on tapes in that coordinate system. Rather than transform the tape information, the vehicle initial conditions and the oblateness accelerations are transformed into the base date system.

A. Vehicle initial conditions that are inserted in an earth referenced system, such as latitude, longitude, altitude, are transformed first to a system determined by the true vernal equinox of date. This system differs from the base date system by the earth's nutation and precession. Transformation by the nutation matrix $[N]$ and the precession matrix $[A]$ thereupon brings the initial conditions into the base date system.

B. The oblateness attraction of the earth is calculated from a knowledge of $\vec{R}$, the position of the vehicle from the center of the earth, expressed in the true earth system. Since vehicle position as calculated in the trajectory portion is in base date components these components must be transformed via precession and nutation into the true earth system. The resultant attraction is then transformed back into the base date system.

The oblateness attraction of the moon is calculated from the vehicle position with respect to the moon's center and the lunar oblateness matrix; the latter takes account of spherical harmonics of potential of degree - 3, based on the three lunar moments of inertia.

2. Definition of Coordinate Systems

The precession, nutation, libration and other transformations that follow all represent rigid rotation of right-handed cartesian coordinate systems; hence the matrices are real, orthogonal and have determinants 1. The coordinate systems involved are defined in this section, the transformations are defined in the following section.

Unit vectors are characterized by a circumflex accent

$\hat{X}_B\ \hat{Y}_B\ \hat{Z}_B$	$\hat{X}_B$ along <u>mean</u> vernal equinox <u>of base date</u> (intersection of ecliptic of base date and mean equatorial plane of base date); $\hat{Z}_B$ normal to mean equatorial plane of base date, positive in northern hemisphere; $\hat{Y}_B$ such that $\hat{X}_B\ \hat{Y}_B\ \hat{Z}_B$ form a right handed system
$\hat{X}_Q\ \hat{Y}_Q\ \hat{Z}_Q$	$\hat{X}_Q$ along <u>mean</u> vernal equinox <u>of date</u> (intersection of mean equator of date and ecliptic of date); $\hat{Z}_Q$ normal to mean equator of date, positive in northern hemisphere; $\hat{Y}_Q$ normal to $\hat{X}_Q$ and $\hat{Z}_Q$ so that $\hat{X}_Q\ \hat{Y}_Q\ \hat{Z}_Q$ form a right handed system.

$\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$

$\hat{X}_E$ along true vernal equinox of date (intersection of true equatorial plane and ecliptic of date);
$\hat{Z}_E$ normal to true equator, positive toward northern hemisphere;
$\hat{Y}_E$ normal to $\hat{X}_E$ $\hat{Z}_E$ and such that $\hat{X}_E$ $\hat{Y}_E$ $\hat{Z}_E$ form a right handed system.

$\hat{X}_M\ \hat{Y}_M\ \hat{Z}_M$

$\hat{X}_M$ along the principal axis of moon, positive on the earth side; $\hat{Z}_M$ along the principal axis of moon, positive in the direction of the rotation of the moon about its axis.
$\hat{Y}_M$ along the principal axis of moon, to form right handed system.

$\hat{X}_G\ \hat{Y}_G\ \hat{Z}_G$

$\hat{X}_G$ in true equator of earth and in Greenwich Meridian; $\hat{Z}_G$ normal to true equator, positive toward north pole; $\hat{Y}_G$ to form right handed system.

$\lambda_E\ \phi_E\ R_E$

Geocentric rt. ascension (apparent siderial time); geocentric angle, or declination of line from earth center to vehicle; geocentric distance from vehicle. See Figure V.

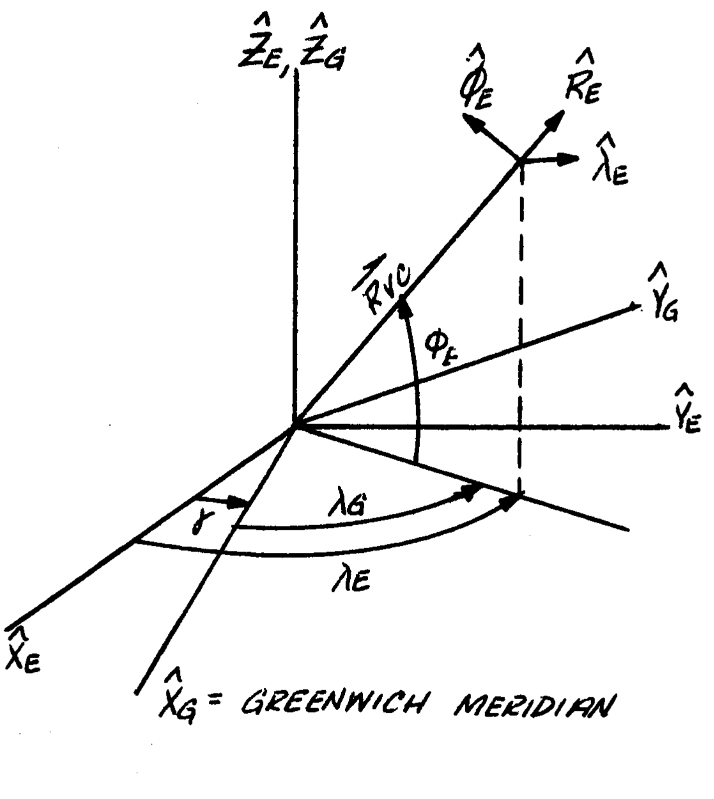


FIGURE V
GEOCENTRIC
COORDINATES

$\hat{X}_E$ true vernal equinox

$\hat{Y}_E$ in true equatorial plane

γ Greenwich hour angle of true vernal equinox of date

$\vec{R}_E = \vec{R}_{VC}$ for earth as dominant body

$\hat{\lambda}_E$ $\hat{\phi}_E$ $\hat{R}_E$

$\hat{\lambda}_E$ normal to vehicle's local meridian, positive eastward; $\hat{R}_E$ along geocentric radius vector to vehicle; $\hat{\phi}_E$ normal to $\hat{\lambda}_E$ and $\hat{R}_E$ and such that $\hat{\lambda}_E$ $\hat{\phi}_E$ $\hat{R}_E$ form a right handed system. See Figure V.

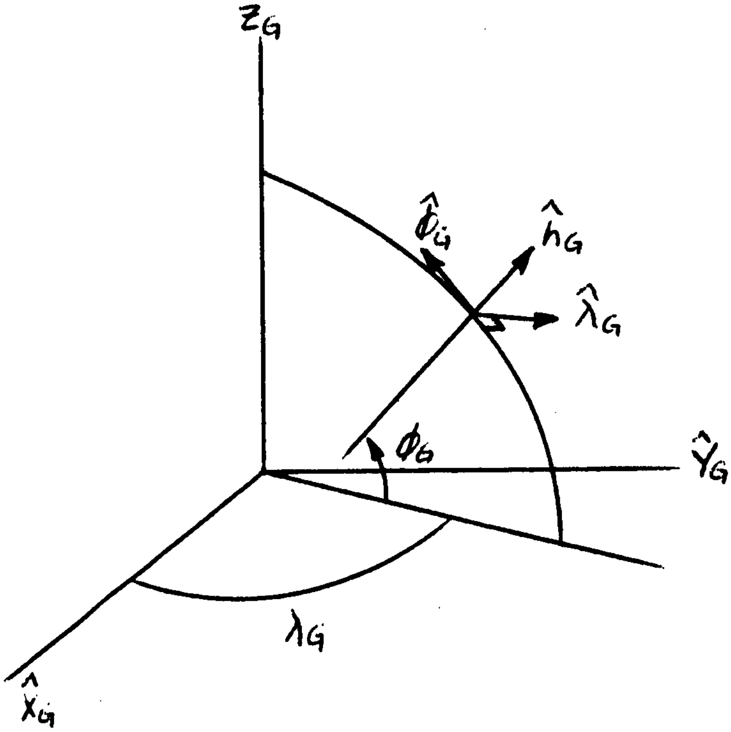
$\hat{\lambda}_G$ $\hat{\phi}_G$ $\hat{h}_G$

local geodetic system; $\hat{\lambda}_G$ normal to meridian, towards east, $(\hat{\lambda}_G = \hat{\lambda}_E)$; $\hat{h}_G$ normal to ellipsoid through position of vehicle; $\hat{\phi}_G$ normal to $\hat{\lambda}_G$ and $\hat{h}_G$ such that $\hat{\lambda}_G$ $\hat{\phi}_G$ $\hat{h}_G$ is right handed. See Figure VI.

λ_G ϕ_G h_G

Geodetic longitude, positive easterly from Greenwich meridian through 360 degrees; geodetic latitude (angle between equatorial plane and normal to ellipsoid through vehicle position); altitude above ellipsoid. See Figure VI.

FIGURE VI
LOCAL GEODETIC COORDINATES



$\hat{\lambda}_M \; \hat{\phi}_M \; \hat{R}_M$ — $\hat{\lambda}_M$ normal to local moon meridian; $\hat{R}_M$ along outward radial from center of moon to vehicle; $\hat{\phi}_M$ normal to $\hat{R}_M$ and $\hat{\lambda}_M$. See Figure VII.

$\lambda_M \; \phi_M \; R_M$ — selenocentric longitude, measured in the $\hat{X}_M \; \hat{Y}_M$ plane in the sense of positive rotation about $\hat{Z}_M$; selenocentric declination; selenocentric distance. See Figure VII.

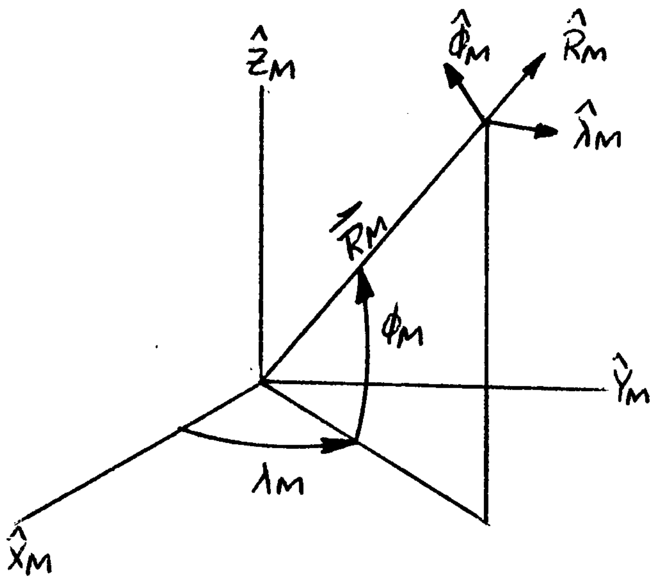


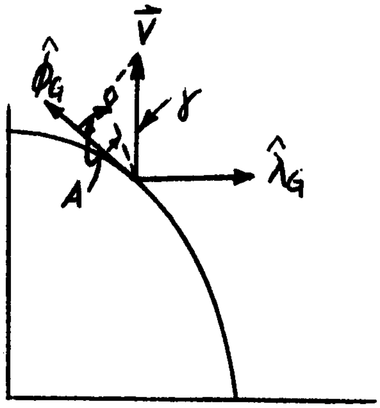
FIGURE VII
SELENOCENTRIC
COORDINATES

$\vec{R}_M = \vec{R}_{VC}$
for moon as dominant body

$V \; A_G \; \gamma_G$ — vehicle speed relative to $X_G \; Y_G \; Z_G$ frame; azimuth, flight path angle relative to $\hat{\lambda}_G, \hat{\phi}_G, \hat{h}_G$ system. See Figure VIII.

A positive CW from north
γ positive up from $\hat{\phi}_G$ $\hat{\lambda}_G$ plane
V always positive, being $|\vec{V}|$

FIGURE VIII
AZIMUTH AND
FLIGHT PATH
ANGLES



V A_E γ_E vehicle speed relative to $\hat{X}_G$ $\hat{Y}_G$ $\hat{Z}_G$ frame, azimuth, flight path angle relative to $\hat{\lambda}_E$ $\hat{\phi}_E$ $\hat{R}_E$ system. See Figure VIII.

3. List of Transformations

The coordinate system under From and To are defined in the previous section of this report. The matrices used are given in Appendix A.

SYMBOL	NAME	ACRONYM	FROM	TO
$[A]$	Precession	PREC	$\hat{X}_Q\ \hat{Y}_Q\ \hat{Z}_Q$	$\hat{X}_B\ \hat{Y}_B\ \hat{Z}_B$
$[N]$	Nutation	NUTA	$\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$	$\hat{X}_Q\ \hat{Y}_Q\ \hat{Z}_Q$
$[L]$	Libration	LIBRA	$\hat{X}_M\ \hat{Y}_M\ \hat{Z}_M$	$\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$
$[\gamma]$	Gamma Matrix	GAMMAT	$\hat{X}_G\ \hat{Y}_G\ \hat{Z}_G$	$\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$
$[G]$	Geodetic to Greenwich Transformation	GENMAT	$\hat{\lambda}_G\ \hat{\phi}_G\ \hat{h}_G$	$\hat{X}_G\ \hat{Y}_G\ \hat{Z}_G$
$[D, RA]$	Declination, Right Ascension	GENMAT	$\hat{\lambda}_E\ \hat{\phi}_E\ \hat{R}_E$	$\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$

Since all the above transformations are orthogonal, the inverse of any is simply its transpose.

4. Transformation of Initial Conditions

The transformation subroutine calls upon other subroutines to convert vehicle initial position and velocity into the base date system of trajectory calculation. Initial conditions consist of three position coordinates and three velocity components. The velocity need not be specified in the same type of coordinates as position.

Table II lists the available options and a brief description of the coordinate system. In the right hand column, the blocks defined in Table A-1 of Appendix A which are required for the particular transformation are listed.

Complete details of these transformations are given in Appendix A.

5. The Effects of [A], [N] and [L] on Oblateness

a. Earth Oblateness

The subroutine OBLATE transforms the vehicle position vector relative to the earth's center into the true earth system $\hat{X}_E \hat{Y}_E \hat{Z}_E$. The oblateness attraction given in equations 6 and 7 is then calculated using the transformed value of $\vec{R}_{VC}$. The resultant attraction vector is then expressed in the base date system. The formulas employed are given in Appendix A.

b. Lunar Oblateness

Circumlunar trajectories are strongly influenced in the neighborhood of the moon by the lunar oblateness mass. Acceleration terms corresponding to lunar oblateness attraction were added as $\vec{F4}$ to equation (1). The method of computing $\vec{F}_4$ is given in Appendix A.

B. <u>Propagation Corrections</u>

1. Introduction

The bending and retardation of radio waves passing through the troposphere and ionosphere limit the inherent precision of modern electronic tracking systems. Some form of refraction correction is therefore necessary to achieve the maximum accuracy of the tracking systems.

Correction of the troposphere error can be approached from two points of view. The first, the analytical method, involves

TABLE II

Option	Load	Table A-1* Blocks
1.	Position in Inertial Cartesian Coordinates Velocity in Cartesian Coordinates referenced to earth's center.	7, 8 $\omega_e = 0$
2.	Position as in 1 Velocity as in case one except referenced to coordinate frame rotating at earth's rate	7, 8 $\omega_e \neq 0$
3.	Position geodetic longitude, latitude and height above sea level. Inertial velocity components along east, north and up coordinates as defined at sub-satellite point.	4, 5 $\omega_e = 0$
4.	Same as option 3, except velocity is along east, north and up coordinates as defined at the sub-satellite point, referenced to coordinate frame rotating at earth's rate	4, 5 $\omega_e \neq 0$
5.	Position as in 3 Speed is magnitude of inertial velocity vector Direction given by flight path angle and azimuth	4, 6 $\omega_e = 0$
6.	Same as option 5 except speed is magnitude of velocity vector referenced to coordinate frame rotating with the earth	4, 6 $\omega_e \neq 0$
7.	Position given by geocentric RA, Decl., distance from earth's center. Inertial velocity along unit vectors defining RA, Decl, and distance	1, 2 $\omega_e = 0$
8.	Same as option 7, except velocity is referenced to coordinate frame rotating at earth's velocity	1, 2 $\omega_e \neq 0$
9.	Position given by geocentric RA, Decl., distance from earth's center. Velocity by flt. pth. angle, az. angle, magnitude of inertial velocity vector	1, 3 $\omega_e = 0$
10.	Same as option 9 except speed is magnitude of velocity vector referenced to coordinate frame rotating with the earth	1, 3 $\omega_e \neq 0$

* Appendix A

assuming a simple exponential decay of the index of refraction with altitude. The resulting tropospheric errors for range and elevation are solvable in closed form as a function of the elevation angle.

The second method is numerical in form. For this method, it is not necessary to assume an exponentially decaying troposphere; any model will suffice. The error is determined by numerically integrating over the total propagation path with the index of refraction at each integration point being determined by the assumed model.

Because of the complex nature of the ionosphere, it is very difficult to find a simple model upon which analytic solutions to the ionospheric errors can be based. Therefore, a numerical approach seems to be more likely for an ionospheric analysis.

In order to be consistent and to achieve higher precision, the numerical approach is used in the program for evaluating both the ionsopheric and tropospheric contributions to the error.

The method that is used is one derived by S.Weisbrod as detailed in Reference 10. The method is particularly simple and can be applied to both tropospheric and ionospheric bending. There are no limitations on the shape of the profile or angle of elevation. The only assumptions are that the index of refraction gradient is only in the vertical plane, that the index of refraction profile can be approximated by a number of linear segments, and that the thickness of these steps is small compared to earth's radius.

These assumptions are readily justifiable in all practical cases. In addition to refractive bending, the problem of signal retardation, resulting in a range error, is considered. Also the effect of ray bending on the range rate error is included.

2. Index of Refraction Models Used

a. General

Since it is impossible to describe atmospheric propagational effects under all parametric conditions, atmospheric models representative of average conditions are employed to simplify the computation problem.

In the above models the following assumptions are made: 1) the troposphere extends to approximately 40 km with refractivity decreasing with height; 2) the region between the top of the troposphere and the bottom of the ionosphere is assumed to have zero refractivity; 3) the ionosphere lies between H_0 and 2000 km; 4) beyond 2000 km. the refractivity is zero.

The formulae used to compute range and elevation errors are, with few exceptions, the same for both the troposphere and ionosphere. The refractivity, however, is computed differently.

This approach will result in answers which are as accurate as the model assumed. Since profiles of refractive index in the atmosphere (especially for the ionosphere) are not precisely known under all conditions a more exact solution seems unwarranted.

b. Tropospheric Model

In this program, the tropospheric model is assumed to be an exponential with the ground index of refraction and the scale height as parameters.

The equation for the refractivity is

$$N = N_0 e^{-h/H} \tag{83}$$

where N_0 = 313 = average refractivity at sea level

H = 7KM = average scale height

$N = (n-1) \times 10^6$, where n is the index of refraction.

c. Ionospheric Model

In the ionosphere, the index of refraction is dependent on more parameters than those considered in the troposphere. As a minimum, the index of refraction in the ionosphere is dependent upon the height of the base of the ionosphere layer, the height of the maximum electronic density of the F2 layer, and the maximum electronic density of the F2 layer. In addition, the index of refraction in the ionosphere is also dependent upon diurnal, solar activity, seasonal, geographical, and daily variations as well as other miscellaneous sporadic variations. Also, unlike the troposphere, the refraction errors in the ionosphere are frequency dependent.

The relationship between the index of refraction, the radio frequency and the electron density in the ionosphere is the following:

$$n = \frac{1 - N_e e^2}{\epsilon_0 m \omega^2} \tag{84}$$

where N_e = electrons per cubic meter

e = electronic charge (1.60×10^{-19} coulomb)

m = electronic mass (9.08×10^{-31} kilogram)

ω = 2π times the frequency

ϵ_0 = permittivity of free space (8.854×10^{-2} farad/meter)

Using the first two terms of the binomial expansion and substituting the above constants, the formula for index of refraction reduces to

$$n = 1 - \frac{40.3\, N_e}{f^2} \qquad 85)$$

This formula holds for frequencies above the critical frequency, f_c, given by the following relationship

$$f_c = 8.97\, N_e^{1/2} \times 10^{-6} \quad mc/s \qquad 86)$$

Defining the refractivity by the following relation

$$N = (n-1) \times 10^6 \qquad 87)$$

equation (85) can be written as follows:

$$N = -4.03\, \frac{N_e}{f^2} \times 10^{-5} \qquad 88)$$

The model used for the electron density versus height profile in the ionosphere consists of a parabolic variation below the height of maximum electron density matched to a hyperbolic secant profile above the maximum.

The relationships are as follows:

$$Ne \begin{cases} = P_o[1-(1-\sigma)^2] & 0 \leq \sigma \leq 1 \\ = P_o \operatorname{sech} \frac{\pi}{4}(\sigma - 1) & \sigma \geq 1 \end{cases} \qquad 89)$$

where Ne = electrons per cubic meter

P_o = maximum density

$$\sigma = \frac{h - Ho}{Hm - Ho} \qquad 90)$$

h = height above the ground

Ho = height of the base of the layer

Hm = height of the maximum electron density

This model has the following preferred characteristics:

1. The model has three degrees of freedom which can be obtained from ionogram data. These parameters uniquely specify the entire distribution.
2. The distribution is parabolic below and immediately above the maximum density, and exponential at great heights.
3. The electron content of the distribution above the maximum is three times that below it.
4. The entire electron density profile and its derivatives are continuous everywhere.

Using this model, the refractive effects of the D and E layers are not singled out because they are quite small in comparison with the effects of the F layer.

d. Computations Employed

A complete discussion of the computation technique employed is given in Appendix C.

C. <u>Data Selection, Ambiguity Resolution and Time Correction</u>

1. The Problem

Prior to the introduction of data into the main MINIVAR program, the available raw data have been passed through edit and merge programs to time order all data from all sources onto one input tape of standard format. (Appendix H describes the data editing and merging programs required prior to the insertion of data into the MINIVAR program.)

There still remain two problems in the data which need resolution prior to use in MINIVAR. They are:

a. The time (which is assigned as the time of an observation) must take into account the finite propagation time from transmission to reception at each end of the link.

b. Some measuring systems produce data which are ambiguous (i.e., the recorded value differs from the real value by some uncertain multiple of a fixed quantity.) The ambiguity is not readily resolvable by the measuring device; it requires some *a priori* data.

2. Method of Handling Problem (See Figure IX)

The program handles these two problems in the following manner:

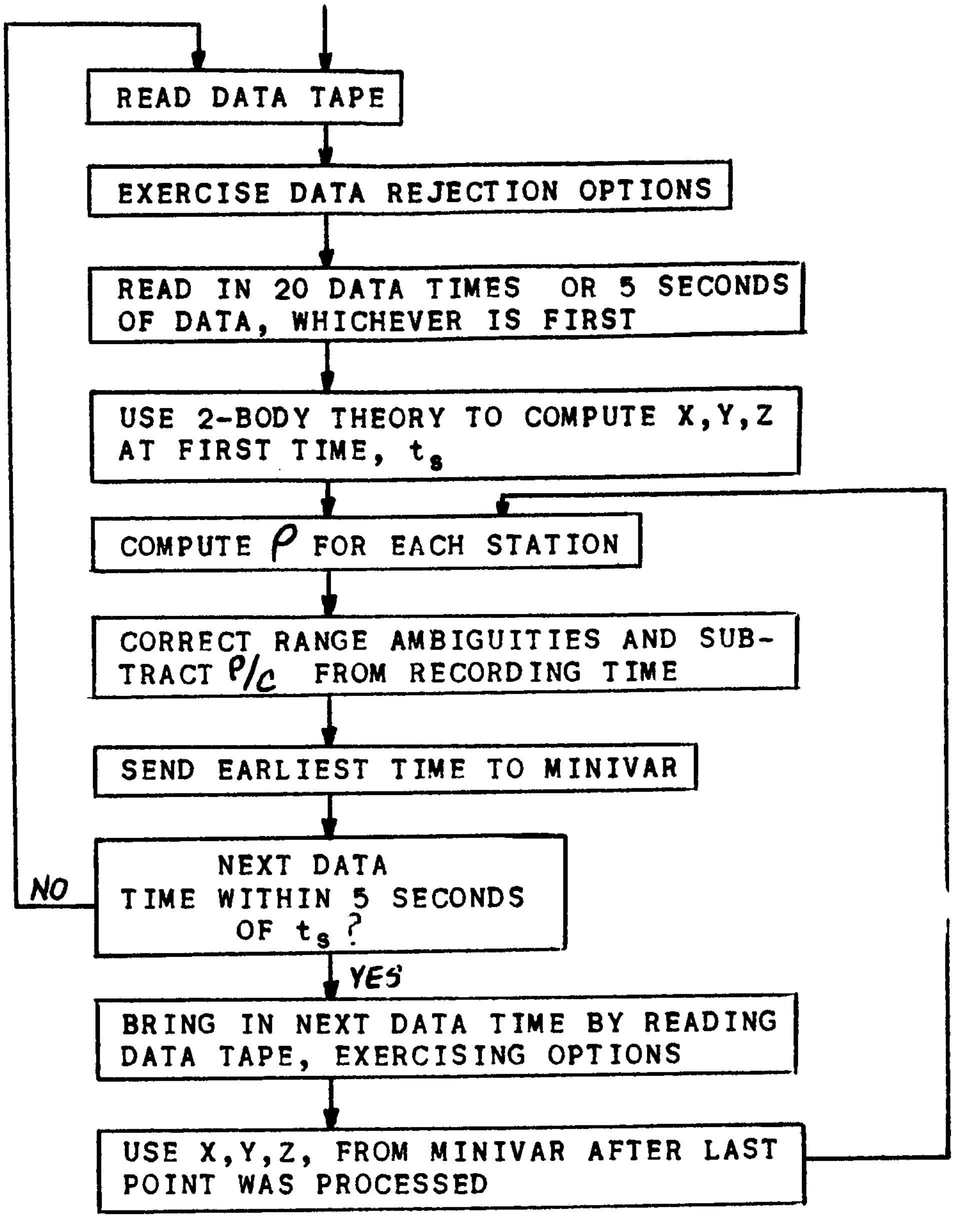


FIGURE IX: DATA SELECTION AND CORRECTION

1. The raw data, already roughly time ordered at the program's input, are read in until either 20 data times or all the data in the first five seconds are read in. Each data time can be made up of to 4 data types from any one station.

2. As each time is read in, the data which are marked as having poor quality are rejected.

3. Options are also available for:

    (a) rejecting data of any type from any station

    (b) rejecting all data from any station or stations

    (c) rejecting any particular data point which is not an integral multiple of an input value. This allows selecting 1n, 2n, 3n, 4n, ... data of any time from any station.

4. The earliest data time in the block is selected, and two body theory computes the vehicle's position at that time, from which the distance ρ (station to vehicle one-way distance) is computed for each station which had data in the block.

5. Those data requiring it have their ambiguities resolved. (See Appendix D for ambiguity resolution in the $R/\dot{R}$ system.)

6. The factor ρ/c (c being the velocity of light) is subtracted from the time recorded for each measurement time, translating back to the time the message was sent from the vehicle.

7. The data are now time-ordered on the basis of their new modified time.

8. The first data point (in time) is sent to the main program, and the Minimum Variance process is used to integrate it into the program.

9. All data times in the block which are identical, within 0.1 millisecond, are processed.

10. A new value of ρ for each station which has data in the block is computed using the new value $X \leftarrow \dot{Z}$ obtained after processing the first data time.

11. With the first data time being used, more data are brought in from the raw data remaining which falls within 5 seconds of the first time (step 4).

Thus, the time correction and ambiguity resolution is continually repeated, the precision increasing as the precision of the orbit improves.

3. Reason for Choice of Data Times And Time Spread

The choice of a maximum of 20 data times in each block stems from the following:

A satellite far removed from the earth could possibly have as many as 10 Minitrack and 2 $R/\dot{R}$ systems taking simultaneous data. Because each station is at a different location, the range is different for each; therefore, the time correction is different for each. Because of the different values of ρ, if say only one data time were brought in at one time, the time correction could possibly put the time of a later point ahead of the point already processed, where is an illegal step in MINIVAR. Therefore, by bringing in 20 points, there remains little possibility of a point being brought in which, after correction, falls ahead of a time already processed.

The choice of a 5 second interval on data stems from the following argument:

Suppose a data point falls once every five seconds. The maximum distance the range ρ might change in the interval would be something like 30,000 ft/sec x 5 seconds = 150,000 feet, or 45.4 KM.

For purposes of time correction, then, the value of ρ could be in error by 45.4 KM, which, when divided by the velocity of light (3×10^5 KM/sec) corresponds to a timing error = 5.1×10^5 seconds or 0.15 milliseconds, an error well within the accuracy of the station timing system and WWV.

D. The $\overline{\epsilon^2}$ Matrix

1. Computed $\overline{\epsilon^2}$

The optimum filter of the Kalman Technique, given by equation 21, contains as an element the _a priori_ estimate of the covariance matrix of the observation, denoted by $\overline{\epsilon^2}$. In the absence of bias errors, assuming actually random distribution of the observation error, and assuming that the only error source would be in the instrument itself, the value used for $\overline{\epsilon^2}$ would be that of the observation instrument.

In this program, the maximum number of observations that can be handled from any one station is four; thus the maximum dimension of the matrix is 4 x 4. The diagonal elements are the variances of the respective observation types, the off-diagonal elements are the covariances. The matrix can be presented in this form, or alternately it can take the form

$$\begin{bmatrix} \sigma_1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \sigma_2 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \sigma_3 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \sigma_4 \end{bmatrix}$$

where

$$\sigma_{1-4} = \sqrt{\text{VARIANCE}_{1-4}}$$

$$\rho_{mn} = \rho_{nm} = \frac{(\text{covariance})_{mn}}{\sigma_m \, \sigma_n}$$

ρ_{mn}, the correlation coefficient, has a value range from -1 to +1.

In the Kalman-Schmidt Filter, if the ρ's are 0, the results will be identical whether the n $\leq$ 4 data types per station are processed as a group or singly. With non-zero ρ's the data would be processed as a group.

2. $\overline{\epsilon^2}$ as a Kalman Filter Inhibitor

When the initial conditions ascribed to the vehicle are in error by a considerable amount, the initial residuals are large and, consequently, large changes to the orbit occur. Under certain unusual situations (such as large gaps in data), these large changes can lead to even larger residuals, which eventually can lead to divergence of the solution.

The program utilizes a constant multiplier times the input value of $\overline{\epsilon^2}$, to effectively reduce the weight given to the data. In this way, the orbit is caused to converge to the true orbit without over correcting.

This multiplier (which is an input quantity) is used on the first iteration through the data, and is reduced to unity for subsequent iterations. This can be done since the first iterative processing of the data should cause the initial conditions used for subsequent iterations to be much better than the original estimate.

Operation

VII. PROGRAM DESCRIPTION

A. General Description of Program Usage

In the "determination mode" the MINIVAR program is used for obtaining an estimate of the position and velocity of a satellite based on accrued tracking data. In order to make this possible, a precise satellite ephemeris computation technique (using Encke's method) is employed along with the matrix manipulations required by the Minimum Variance method. These have been described in some detail in Sections III and IV. In addition, the available tracking data must be corrected for propagation anomalies and errors in ground station location. Also, certain ambiguities in the tracking data must be resolved.

The equations of motion are given with respect to the earth as the reference body. The perturbing influence of the first three zonal harmonics of the earth, air drag, the oblate moon, and the planets Venus, Mars and Jupiter are included. The positions of the astronomical bodies are provided by an ephemeris tape based on Naval Observatory data. The effect of nutation, precession and libration on the gravitational attraction of the earth and moon are computed.

All computations within the program are in units of earth radii and hours; inputs and outputs are in various units as described in a later section.

The program is designed to handle data from a variety of tracking systems (described in the appendices), and provisions are included for expansion of the program to include new observation types. The data must be time ordered to be consistent with the formulation of the Kalman technique. Thus, the raw tracking data from each tracking system must be edited, time ordered, and merged with data from other stations prior to its insertion into this program. A standard format is employed in this program; any editing programs must have their output formats consistent with the data input to this program.

A number of options are available for excluding portions of the program which tend to increase program precision but which are not obsolutely necessary to its operation. These options include propagation correction, precession, nutation, and libration transformations, and perturbing bodies.

Another important option is the ability to generate data corresponding to any selected orbit. The computed observables, without noise added, are placed on a data tape, the program is then used in the "determination mode" just as with real data. In the determination mode, noise can be added to the computed observables. The magnitude of the standard deviation of the added noise is selectable by the user. This mode is called the "reference mode".

The principal output from the program is a time history of the position and velocity of the vehicle; the time history of the osculating elements, and the time history of the covariance matrix of the position and velocity of the vehicle, either from real tracking data or from data generated internally.

B. Summary of Options Available to the User

A number of options and choices of input conditions are available to the user. The major options and the selections under each are given here in order to give the user some feeling for the choices available. More precise definitions and suggested values are given in subsequent sections. The exact form of the input required is given in Table III.

Reference Mode

A. Purpose

To generate a set of time observations of a satellite orbit from selected topocentric positions and to store them so that they can be used in test runs in the "determination" mode.

B. Options

	Symbol or Location (See Table III)
a. Orbit initial conditions	
1. Coordinate System	KLM1
2. Units of vector	KLM
3. Components of Vector	RCIN(1-3) RDCIN(1-3)
4. Time of Initial Conditions	NYEARP
	DAYS
	HRS
	HMIN
	SEC

b. Integration, print and measurement intervals

1. Integration interval, near earth — DTNE

2. Integration interval, far earth — DTFE

3. Print and measurement interval, near earth — PRNTNE

4. Print and measurement interval, far earth — PRNTFE

c. <u>Perturbing sources</u>

1. use or reject, individually, the perturbing influences of the Sun, Moon, Venus, Mars, Jupiter — BMU

2. drag coefficient — CDRAG

3. vehicle area to mass ratio — AMASS

d. <u>Total time of run</u> — TMAX

e. <u>Station Information</u>

1. Station locations (maximum= 25)

   geodetic latitude — SLAT, SLATM, SLATS

   geodetic longitude — SLON, SLONM, SLONS

   height above reference geoid — SALT

   correction for geodetic net error $(\Delta u, \Delta v, \Delta w)$ — TEBAR

   Coordinate system rotation, $R/\dot{R}$ system, α — ROTXY(K)

2. Radio frequencies (for propagation calc.)

   transmitter — FUP

   receiver — FDWN

3. Data availability

   data available at each station — TEBAR

   data to be generated by program at station if available(indicated by TEBAR) — ITSAVE

f. <u>Corrections</u>

1. Compute precession, nutation and libration effects	KLM2
2. Compute refraction corrections	KRF
ground index of refraction	XNZ
tropospheric scale factor	HALT
integration step size for tropospheric evaluation	DH1
upper limit to troposphere	H2
maximum electron density	PZERO
height of bottom of ionosphere	HZERO
height of max. electron density	HM
upper limit of ionosphere	H4
integration step size for ionosphere evaluation	DH2

g. <u>Output Options</u>

See Section VIII-C.

Determination Mode

A. Purpose

To take real data (or data generated in the reference mode) and determine the orbit of the vehicle.

B. Options

a. Estimate of orbit initial conditions	Symbol or location
1. Coordinate system	KLM1
2. Units of vector	KLM
3. Components of vector	RCIN(1-3)
	RDCIN(1-3)

4. Time of initial conditions — NYEARP, DAYS, HRS, HMIN, SEC

5. Covariance matrix of estimate — PMAT

b. <u>Integration, print and measurement intervals</u>

1. Integration interval, near earth — DTNE

2. Integration interval, far earth — DTFE

3. Print interval, near earth — PRNTNE

4. Print interval, far earth — PRNTFE

c. <u>Perturbing sources</u>

1. Use or reject, individually, the perturbing influences of the Sun, Moon, Venus, Mars, Jupiter — BMU

2. Drag coefficient — CDRAG

3. Vehicle area to mass ratio — AMASS

d. <u>Timing and iteration</u>

1. Length of time of iterations — TMAX

2. Number of iterations in first and second passes — ITERS

3. Length of time in second pass — TMAX2

e. <u>Station information</u>

1. Station locations (maximum=25)

geodetic latitude	SLAT, SLATM, SLATS
geodetic longitude	SLON, SLONM, SLONS
height above reference geoid	SALT
coordinate system rotation($R/\dot{R}$ system)	ROTXY(K)
correction for geodetic net error (Δu, Δv, Δw)	TEBAR

2. Radio frequencies (for propagation calculation)

transmitter	FUP
receiver	FDWN

3. Data selection and rejection

Data types at each station	TEBAR
Save data of this type	ITSAVE
Rejection of data of any given type at every Kth data point	NUM(1-16)
Reject data beyond K σ at first iteration and C σ on subsequent iterations	IRDATA
Magnitude of K	WSW(1)
Magnitude of C	WSW(2)

4. Measurement System Accuracy

standard deviation of measurement, σ	TEBAR
correlation coefficient, ρ	TEBAR

f. <u>Improvement of Precision</u>

1. Add terms to input $\overline{\epsilon^2}$ — CMLT

variance of station location ($S2u$, $S2v$, $S2w$)	TEBAR
variance of timing error	SDT
variance of speed of light	SVL
variance of maximum electron density	SRM
variance of ground index of refraction	SN1
variance of GM (earth)	SGM
variance of GM (moon)	SMM
variance of second harmonic, J_{20}	SJ_2

variance of third harmonic, J_{30}	SJ_3
variance of fourth harmonic, J_{40}	SJ_4

g. Noise Generation

1. Add error to observables with standard deviation equal to $\overline{\epsilon^2}$ (should be used only with data generated in reference mode)	TADD

h. Output Options

See Section VIII-C

VIII. OPERATING PROCEDURES

A. General

The program is run as a standard Monitor job. The program, followed by the Specifications Input (described in Section VII-B.1.) is loaded on physical tape A-2. Standard (off-line) output is written onto physical tape A-3. The tape containing ephemeris tables is mounted on physical tape A-5.

In the reference mode, a blank tape is mounted on logical tape 16 and the observations generated are written onto this tape in binary format. For the determination mode, the binary tape containing the observations to be used in the orbit determination is mounted on logical tape 16. Other tapes required for optional outputs are described in a subsequent section.

Several cases can be run as one Monitor job by stacking the appropriate sets of Specifications Input behind the program on A-2. At the start of execution of each case a program pause occurs and an on-line message, "PAUSE TO MOUNT TAPES," is printed. This allows for running successive cases which may require different observation data tapes. After mounting is completed, execution is resumed by depressing the START key on the operator's console.

B. Input

1. Specifications Input

This is the input which is loaded behind the program and contains initial conditions, case parameters, station information and control options.

a. Format

A detailed description of the INPUT Format is given by Table III. The data are entered in twelve sections. The data for each section are preceded by a heading card with the section number entered in columns 1-5 as an integer. The quantity in the description column is entered on the specified card of the section in the appropriate columns. The name given is the name used for the quantity in the program.

TABLE III: SPECIFICATIONS INPUT

Sect.	Card	Cols.	Name	Type	Description
1	1	2-72	ITITLE	12A6	Title
2	1	1-18	TMAX	E18.8	Final time, hrs.
		19-36	TMAX2	E18.8	Final time of second run, hrs.
	2	1-18	DTNE	E18.8	Near earth integration interval, hrs.
		19-36	DTFE	E18.8	Far earth integration interval, hrs.
		37-54	PRNTNE	E18.8	*Near earth print interval,hrs.
		55-72	PRNTFE	E18.8	*Far earth print interval,hrs.
	3	1-5	NYEARP	I5	Year of initial conditions
		6-10	DAYS	F5.0	Day of year (Jan. 1 is day 1)
		11-15	HRS	F5.0,	Hour of day
		16-20	HMIN	F5.0	Minute of hour
		21-30	SEC	F10.0	Second of minute
3	1	1-6	BMU	6F1.0	6 values - represent attracting bodies - if body is not used, insert 0, if used insert 1 in the array - left to right 1 Earth 2 Sun 3 Moon 4 Venus 5 Mars 6 Jupiter
		7-10	KLM	I4	Indicator for input dimensions 0 for earth radii, ER/hr 1 for km, km/sec

*In reference mode, observations are generated at this time.

Sect.	Card	Cols.	Name	Type	Description
3 cont		11-15	KLM1	I5	Type of input coordinates 1 Cartesian coord. $\omega_e = 0$ 2 Cartesian coord. compute ω_e 3 Geodetic long,lat,ht; V_e, V_n, V_h; $\omega_e = 0$ 4 Geodetic long,lat,ht; V_e, V_n, V_h; compute ω_e 5 Geodetic long,lat,ht; $\lvert V\rvert, \gamma, Az$; $\omega_e = 0$ 6 Geodetic long,lat,ht; $\lvert V\rvert, \gamma, Az$; compute ω_e 7 Geocentric RA,DEC ,Ht; V_{RA}, V_{DEC}, V_{HT}; $\omega_e = 0$ 8 Geocentric RA,DEC ,Ht; V_{RA}, V_{DEC}, V_{HT} compute ω_e 9 Geocentric RA,DEC ,Ht; $\lvert V\rvert, \gamma, A_z$; $\omega_e = 0$ 10 Geocentric RA,DEC ,Ht; $\lvert V\rvert, \gamma, Az$; compute ω_e
	1	16-20	KLM2	I5	Flag for precession & nutation 1 for prec. & nuta., input vectors in true earth system 0 no prec. & nuta 2 for prec. & nuta., input vectors in base date system
		21-25	KLM3	I5	Data time correction flag 0 to bypass 1 to compute correction
		26-30	KRF	I5	Refraction correction flag- 0 to bypass 1 to compute corrections
		31-35	KDATA	I5	Number of data points in determination mode, zero in reference mode
		36-40	KPRTR	I5	If non-zero a time history is generated on logical tape 18. See I018 for other use of L.T. 18.

Sect.	Card	Cols.	Name	Type	Description
3 cont.		41-45	IFLAGS	I5	Print control for Kalman calc.- -1 for every time 0 to bypass 1 approximate print interval, mins.
		46-50	IOBS	I5	Flag to compute observations 0 for no observations 1 for observation-no summary 2 for observation and summary
		51-55	KONDS	I5	Print control for additional data (see IFLAGS)
		56-60	ITERS	I5	Number of forward iterations t_o to TMAX and t_o to TMAX2
		61-65	IRDATA	I5	Flag to reject K σ data (first iteration) and C σ data (subsequent iterations (WSW(1),WSW(2)
		66-70	IØ18	I5	Print BCD observables on LT18; if non-zero KPRTR must be zero
4	1	1-18	CDRAG	E18.8	Drag coefficient
		19-36	AMASS	E18.8	Area/mass - cm^2/gm
	2	1-72	WSW	4E18.8	(1) input value is number of σ's above which data are rejected on first pass(See IRDADA) (2) input value is number of σ's above which data are rejected on subsequent iterations
	3	1-30	1WSW	6I5	(1) = 0, use grown P_o on iterations $\neq$ 0, use initial P_o on iterations Others not used
5	1	1-18	VNAME	3A6	Name of vehicle
		19-36	TADD	E18.8	Initial random number; zero if no noise is to be added to data
		43-54	ITSAVE	012	Data types to be generated in reference mode.

Sect.	Card	Cols.	Name	Type	Description
5 cont	1 cont	43-54 cont			0 No data of this type 1 data of this type types 1-12 from right to left In Determination, indicates data types to be processed if desired and if available.
6	1	1-70	NUM(1-14)	14I5	Data selection indicator
	2	1-10	NUM(15-16)	2I5	=0 Bypass this type =K Use every Kth of this type
	3-6	1-72	TIME(I)	4E18.8	Time of first acceptance for every Ith type
7	1	1-18	RCIN(1)	3E18.8	Initial position vector(See KLM1)
		19-36	RCIN(2)		
		37-54	RCIN(3)		
	2	1-18	RDCIN(1)	3E18.8	Initial velocity vector(See KLM1)
		19-36	RDCIN(2)		
		37-54	RDCIN(3)		
8	1	1-5	NUMSTA	I5	Number of stations input (maximum of 25)
	2	1-3	K	I3	Station number
		4-15	STANM	2A6	Station name
		16-20	TEBAR(5,1,K)	F5.0	Data types available at station <u>in ascending</u> order with trailing zeroes if less than four types.
		21-25	TEBAR(5,2,K)	F5.0	
		26-30	TEBAR(5,3,K)	F5.0	
		34-35	TEBAR(5,4,K)	F5.0	
		37-54	FUP(K)	E18.8	Transmitter frequency(megacycles/sec)
		55-72	FDWN(K)	E18.8	Receiver frequency (megacycles/sec)

Sect.	Card	Cols.	Name	Type	Description
8 cont	3	1-5	SLON	F5.0	Longitude,deg. (positive east) geodetic
		6-10	SLONM	F5.0	Long., min.
		11-20	SLONS	F10.0	Long., sec.
		21-25	SLAT	F5.0	Latitude, deg.(positive north) geodetic
		26-30	SLATM	F5.0	Lat., min.
		31-40	SLATS	F10.0	Lat., sec
		41-50	SALT	F10.0	Geodetic altitude, ft.
		51-72	ROTXY(K)	E22.8	Rotation angle of $\dot{R}/R$ stations (α), radians
	4	1-18	TEBAR(1,1,K)	4E18.8	*Standard deviation of data type specified by TEBAR(5,1,K) (Card 2)
		19-36	TEBAR(1,2,K)		*Correlation coefficient between TEBAR(1,1,K) and TEBAR (2,1,K) (Card 2)
		37-54	TEBAR(1,3,K)		*Correlation coefficient between TEBAR(1,1,K) and TEBAR(3,1,K) (Card 2)
		55-72	TEBAR(1,4,K)		*Correlation coefficient between TEBAR(1,1,K) and TEBAR(4,1,K) (Card 2)
	5	1-18	TEBAR(2,1,K)	4E18.8	S2U, Variance of geodetic net error, u direction, $(ER)^2$
		19-36	TEBAR(2,2,K)		σ_{22} See σ_{11}
		37-54	TEBAR(2,3,K)		ρ_{23} See ρ_{13}
		55-72	TEBAR(2,4,K)		ρ_{24} See ρ_{14}

*See next section for units for various observation types

Sect.	Card	Cols.	Name	Type	Description
8 cont	6	1-18	TEBAR(3,1,K)	4E18.8	S2V, Variance of geodetic net errors, v direction, $(ER)^2$
		19-36	TEBAR(3,2,K)		S2W, Variance of geodetic net error, w direction, $(ER)^2$
		37-54	TEBAR(3,3,K)		σ_{33}, See σ_{11}
		55-72	TEBAR(3,4,K)		ρ_{34}, See ρ_{14}
		1-18	TEBAR(4,1,K)	4E18.8	Δu, Station location error geodetic net error, u direction (ER)
		19-36	TEBAR(4,2,K)		Δv, Station location error caused by geodetic net error, v direction (ER)
		37-54	TEBAR(4,3,K)		Δw, Station location error caused by geodetic net error, w direction (ER)
		55-72	TEBAR(4,4,K)		σ_{44}, See σ_{11}

repeat cards 2-7 for each station

Sect.	Card	Cols.	Name	Type	Description
9	1-6	1-12	PMAT(6,6)	6E12.8	(6x6) matrix - initial
		13-24			estimate of the covariance
		25-36			matrix. Each card contains
		37-48			one row of the matrix
		49-60			Units of diagonal elements are $(ER)^2$ and $(ER/HR)^2$. Units of off-diagonal elements $(ER)^2$, ER^2/HR and $(ER/HR)^2$.
		61-72			
10	1	1-18	CMLT	E18.8	0-bypass computations synthesizing $\bar{\epsilon}^2$
		19-36	SRM	E18.8	$(electrons/m^3)^2$, variance of maximum electron density

Sect.	Card	Cols.	Name	Type	Description
10 contl	cont	37-54	SDT	E18.8	HRS^2, variance of timing error
		55-72	SVL	E18.8	$(ER/HR^2)^2$, variance of speed of light
	2	1-18	SGM	E18.8	$(ER^3/HR^2)^2$, variance of GM
		19-36	SMM	E18.8	$(ER^3/HR^2)^2$, variance of moon's gravitational attraction
		37-54	SJ2	E18.8	(unitless), variance of second harmonic
		55-72	SJ3	E18.8	(unitless), variance of third harmonic
	3	1-18	SJ4	E18.8	(unitless), variance of fourth harmonic
		19-36	SN1	E18.8	(unitless), variance of ground index of refraction
		37-54	SN2	E18.8	Multiplier times input matrix used on first iteration
	4	1-18	PEEK	E18.8	Sampling gross interval
		19-36	SAMPLE	E18.8	Sampling period
		37-54	RATE	E18.8	Sampling step size
11	1	1-18	PZERO	E18.8	Maximum electron density, el/m^3
		19-36	XNZ	E18.8	Refractivity at station, unitless
		37-54	HALT	E18.8	Tropospheric Model scale factor, KM
		55-72	DH1	E18.8	Troposphere integration step size, KM
	2	1-18	DH2	E18.8	Ionosphere integration step size, KM.
		19-36	HZERO	E18.8	Height of bottom of ionosphere-KM
		37-54	HM	E18.8	Height of max electron density - KM

Sect.	Card	Cols.	Name	Type	Description
11 cont	2 cont	55-72	H2	E18.8	Upper limit of Troposphere,KM
	3	1-18	H4	E18.8	Upper limit of Ionosphere,KM
12					End of input

b. Further Descriptions and Input Values

Where required, this section will describe, in some detail, the input quantities and typical values to be used. The list will be in the order of the input list of Table III.

TMAX — This time, in hours, represents the total ephemeris time of interest, measured from the time of the initial conditions. When using the iterative mode, it represents the maximum time of the first n (= ITERS) iterations.

TMAX2 — After ITERS iterations to TMAX, the program can change TMAX to TMAX2 and repeat through ITERS iterations to TMAX2.

DTNE — This is the nominal integration interval used when the vehicle is within 4 earth radii. Its value can range from .03125 to .25 hrs. with .0625 hrs. being a reasonable nominal value.

DTFE — The integration interval used when the vehicle is beyond 4 earth radii. Its value can range from .125 to 1.0 with 0.25 hrs. being a reasonable nominal value.

KLM1 — More complete definitions of the input coordinate systems are given in Section VI of this manual.

KLM2, KRF, KLM3 — Flags for computing prec/nuta, refraction and time corrections. Their use with real data improves precision at the expense of added running time.

ITERS — In determination mode, program will iterate through data this many times to TMAX, then, this many times again to TMAX2 if it is greater than TMAX.

IRDATA   Rejects data whose residual is $> K\sigma$ on first iteration and $> C\sigma$ on subsequent iterations (acts the same when TMAX or TMAX2 is the maximum time).

C is input as WSW(2)

K is input as WSW(1)

CDRAG   Drag coefficient to be used when low satellites are being considered. Nominal value is 0.1

AMASS   Area/Mass ratio of satellite. Value can be computed from AMASS =

$$\frac{\text{Frontal area (ft}^2\text{)} \times 2.0481}{\text{Satellite weight (lbs)}} = \frac{\text{area (in cm}^2\text{)}}{\text{mass (in grams)}}$$

, the required input dimensions

TADD   See subroutine description ANDRN, where TADD corresponds to X. Floating point 1.0 is a reasonable nominal value. Necessary to add noise to errorless observations generated in reference mode.

ITSAVE   Indicator for data to be generated in reference mode or to be processed in determination mode. This, plus indicator for data available from each station (in TEBAR), are both required to generate or process data of a particular type from a particular station.

NUM(1-16)   Indicator which allows the bypassing (of the data type indicated) of all but the Kth time the data is available. This gives the user a method of rejecting available data at periodic intervals.

TEBAR(5,1,K) (5,2,K) (5,3,K) (5,4,K)	Define the types of measurements available at each station, 4 maximum
FUP(K), FDWN(K)	Transmitter and receiver frequencies of station
ROTXY	Rotation angle, α , as defined in Figure IV. (R & $\dot{R}$ system only.)

TEBAR The matrix input is

$$\begin{vmatrix} \sigma_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ S2U & \sigma_{22} & \rho_{23} & \rho_{24} \\ S2V & S2W & \sigma_{33} & \rho_{34} \\ \Delta U & \Delta V & \Delta W & \sigma_{44} \end{vmatrix}$$

In operation, the 6 lower left elements are removed for use in their proper places, and elements m, n are overwritten by the quantities in n, m. The m = n elements are squared and the m $\neq$ n elements are overwritten with $\rho_{mn}\ \sigma_m\ \sigma_n$.

Units are:

σ = earth radii, earth radii/hr, radians, direction cosine units

ρ - unitless

Typical values are given in appendices for particular measurement systems.

S2U, S2V, S2W Variance of station location in u,v,w coordinate system (see Appendix K for typical values). Units are (earth radii)^2.

ΔU, ΔV, ΔW	Corrections to station location in u,v,w coordinate systems. See Appendix K for typical values. Units are ER.
PMAT(6,6)	Covariance matrix of estimate of the orbit's initial conditions. Diagonal elements are variances of $X \rightarrow \dot{Z}$, consecutively down the diagonal. Units are $(ER)^2$ and $(ER/HR)^2$. Off-diagonal elements are corresponding covariances, units of $(ER)^2$, ER^2/HR, and $(ER/HR)^2$. Typical input matrix is

$$\begin{vmatrix} 10^{-7} & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^{-7} & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^{-7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^{-6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-6} \end{vmatrix}$$

or

$$\sigma_x = \sigma_y = \sigma_z = 2\,KM$$

$$\sigma_{\dot{x}} = \sigma_{\dot{y}} = \sigma_{\dot{z}} = 1.77\,m/s$$

SRM	variance of electron density at maximum point in ionosphere. Typical value is 81×10^{22} $(el/m^3)^2$.
SDT	variance of timing error of data. Typical value is 1 $(\text{millisecond})^2 \doteq 7.716 \times 10^{-14}$ $(HRS)^2$
SVL	variance of speed of light Typical value is $(1 \times 10^{-6} \times c)^2 = .02856$ $(ER/HR)^2$

SGM variance of the product of the universal gravitational constant and the earth's mass.

Typical value is 2.245×10^{-8} $(ER^3/HR^2)^2$

SMM variance of the product of the universal gravitational constant and the moon's mass

Typical value is 5.993×10^{-12} $(ER^3/HR^2)^2$

SJ2 variance of second harmonic of earth's potential

Typical value is 4×10^{-14} (no units)

SJ3 variance of third harmonic of earth's potential

Typical value is 4×10^{-16} (no units)

SJ4 variance of fourth harmonic of earth's potential

Typical value is 1×10^{-14} (no units)

SN1 Variance of index of refraction at ground

Typical value is 400 (no units)

SN2 This is a multiplier times the $\overline{\epsilon^2}$ matrix used on the first iteration to TMAX and TMAX2.

Typical values are 10^2 to 10^6.

PEEK, **SAMPLE**, **RATE** See section VIII-C.

PZERO Maximum electron density

Typical value is 10^{12} el/m^3

XNZ Refractivity at station

Typical value is 313 (unitless)

HALT Scale factor of tropospheric model

Typical value is 7KM

DH1 Step size in computation of tropospheric error

Typical value is 1KM

DH2 Step size of the integration in the ionospheric error

Typical value is 10KM

HZERO Height of the bottom layer of the ionosphere

Typical value is 250 KM

HM Height of the peak electron density of the ioneephere

Typical value is 400 KM

HZ Upper limit of troposphere

Typical value is 40 KM

H4 Upper limit of ionosphere

Typical value is 2000 KM

2. Observation Data Input

a. General

The observation data are supplied on tape.

The format for the tape is described below. It should be noted that the order of the observations cannot be violated, i.e.,

1) Azimuth
2) Elevation
3) Range
4) Range Rate
5) Right Ascension
6) Declination
7) ℓ Direction Cosine

8) m Direction Cosine

9) X Antenna Angle

10) Y Antenna Angle

11) Δt (Range Equivalent)

12) $\Delta t'$ (Range Rate Equivalent)

13)-16) Open

The program is, at present, limited to 16 types of observations.

b. Tape Format

A binary tape containing the observation data must be supplied with the following format:

Each record contains 21 words

WORD 1	Integral hours since 1960 Jan 1,0^h to time of observation
WORD 2	Integral minutes beyond hours
WORD 3	Integral and fractional seconds beyond minutes
WORD 4	Station number (same as one assigned in section 8 of the Specification Input)
WORD 5	Observation type word (same format as ITSAVE of section 5 of the Specification Input)
WORD 6 - WORD 21	The actual observation values. There are as many values as there are non-zero digits of WORD 5. These values are packed in the lower order words while trailing zeros fill the remaining words of the total 16. The only exception to this trailing zero fill is when real Range-Range Rate data are used (see Appendix I).

Angles are in radians; distance in earth radii, rates in earth radii/hr and time measurements in seconds.

Note: In the reference mode, this format is produced on logical tape 16 by the program for use in the determination mode on logical tape 16.

C. Output

1. Standard (Off-Line) Output

This output is divided into three separate parts, the frequency of output of each part being controlled by a value read in with the Specifications Input. The names of all quantities being written out are specified along with the numerical value. When the program is operating in the reference mode, the quantities which are not pertinent are deleted. The quantities included in each part and the manner of controlling frequency of output is now given.

SECTION 1: This section is output at intervals of PRNTNE when in near-earth reference and at intervals of PRNTFE when in far-earth reference. If the program is operating in the determination mode, this section is automatically output at every observation time point. The quantities output are the time, the components and magnitudes of RC and RDC (instantaneous position and velocity vectors, respectively) and $\overrightarrow{\Delta X}$ (the correction vector to the state variables, in earth radii and earth radii/hour). The dimensions of RC and RDC are determined by the input quantity KLM.

SECTION 2: The frequency of the output of this section is controlled by the input quantity KONDS in the following way:

If KONDS = -1, this section is output every time section 1 is output. If KONDS is 0, this section is never output. If KONDS = N, then this section is output approximately every n minutes. The quantities output in this section during the determination mode are the same as section 1 plus:

a) the osculating elements and astronomical information

b) the station name, observed data, computed data and the difference between them

c) the M matrix for the data observed

d) the $\overline{\epsilon^2}$ matrix employed at that data point

e) the P matrix after the data have been included.

If the running program time is not equal to a particular observation data time, only parts a) and b) will be printed in addition to Section 1 output. In this case, the observed data printed are that of the next observation time which the program will encounter.

In the reference mode only Section 1 output and parts a) and b) are printed. In this case, the observed value will be zero but the computed value will be that associated with the present program time. In the determination mode, this computed value will be observed data.

SECTION 3: The frequency of output of this section is controlled by IFLAGS in the same way that section 2 was controlled by KONDS. The output in this section consists of the matrices involved in the minimum variance filter, in dimensions of earth radii and earth radii/hour. The matrices are S, $Q(t_{i-1})$, $\psi(t, t_r)$, $\psi(t, t_o)$, $Q^-(t_i)$, N, NQN*, $\overline{\epsilon^2}$, Y, Y^{-1}, L, LNQ, and $Q*(t_i)$ and $P^+(t_i)$.

If a summary has been called for in the Specifications Input, it will be output at the end of each case. The quantities output are: Time of each observation, observed value and its residual for each observation type; root-mean-square value of the residuals for each observation type. All dimensions are labeled on the output.

2. Additional (Optional) Off-Line Output

If a summary has been specified, a blank tape must be mounted on logical tape drive 17. This tape is automatically rewound at the completion of each case, so that a new tape does not have to be mounted at the pause for the beginning of a new case.

If the quantity IO18 has been set to 1 in the Specifications Input, a blank tape should be mounted on logical tape 18. At the completion of a case, this tape will contain, in BCD form, the data used with time correction and range ambiguity resolution in packed form with Station Identification and time tags.

It is possible to obtain a time history of the trajectory, instead of the preceding information, on logical tape 18. This is accomplished by setting KPRTR to non-zero in the Specifications Input.

To obtain additional standard printout at periodic intervals over and above the normal print intervals, the user can specify a PEEK, SAMPLE and RATE. Printout will occur every PEEK hours for a period of SAMPLE hours at an interval of every RATE hours. For example if PEEK = 5, SAMPLE = 1, RATE = 0.5, the user will obtain every 5 hours a standard printout (as specified above)for a period of 1 hour once every 30 minutes. Therefore, there will be 10 additional printouts.

3. Optional On-Line Output

If Sense Switch 4 is down, the following quantities are output on the on-line printer during the running of the case:

i. the value of each observation, its residual, and the running root-mean square value of the residual;

ii. the diagonal elements of the P matrix in input units;

iii. the diagonal of the Keplerian parameter covariance matrix. (See Subroutine SOMEGA)

Program

IX. DIRECTORY OF IMPORTANT PROGRAM SYMBOLS

A. Alphabetical List of Symbols in Common

This is a complete list of the quantities in common. This list covers the important and most used FORTRAN mnemonics. It should serve as a guide to help in making changes or additions to the program.

A	Semi-major axis of two-body solution
AABS	Magnitude of A
ACC	Simulated accumulator for integration scheme
ALAMDA	Parameter transition matrix
ALMAT	L matrix of minimum variance filter
AMASS	Ratio of area/mass
AMMAT	M matrix
AREF	Reference value of A
AUERAD	Conversion factor for A.U. to E.R.
BMASK	Octal flags for data types
BMU	Array of working body indicators
CA CB CC CD CE CF	Parameters pertinent to rectification
CDRAG	Drag coefficient
CKMER	Conversion factor for E.R. to km.

CKSERH	Conversion factor for E.R./hr to km/sec
CLAT	Cosine of latitude
CLUE	Logical indicator
CMLT	Indicator for computation of additions to elements of $\overline{\epsilon}^2$ matrix
COMG	Ω of oscubting elements
CONA	2nd gravitational harmonic coefficient
CONJ	1st gravitational harmonic coefficient
CONK	3rd gravitational harmonic coefficient
CPOS	Block of reference body positions
CRAD	Conversion factor from degrees to radians
CWLIN	Block of perturbation values and their 1st and 2nd derivatives
DAYK	Number of days since epoch
DAYS	Launch day of year
DELALP	Array of corrections to alpha parameters
DELTA	Correction to elevation angle
DELX	Array of corrections to state variables
DELY	Array of residuals (observed value-computed value)
DFRHO	Linearization of $d\rho/dh$
DH1	Refraction increment in troposphere
DH2	Refraction increment in ionosphere
DTFE	Far-earth integration step-size
DTI	Current integration step-size
DTNE	Near-earth integration step-size
EBAB	Covariance matrix of observations

ECC	Eccentricity of orbit
EI	Inclination
EN	Mean motion
EPSSQ	Square of earth's ellipticity
ERAD	Earth's radius in km
F1, F2	Up and down frequencies of tracking signal
FA	f_1 term in series expansion of Keplerian Model
FB	f_2 term in series expansion of Keplerian Model
FC	f_3 term in series expansion of Keplerian Model
FD	f_4 term in series expansion of Keplerian Model
FDOWN	Array of station receiver frequencies
FPTH	$F'(\Theta)$ term used in Keplerian model
FRHO	Array of atmospheric densities
FTH	$F(\Theta)$ term used in Keplerian Model
FUP	Array of station transmitter frequencies
HAFPI	90° in radians
HALT	Scale factor for tropospheric refraction model
HM	Altitude of ionospheric maximum electron density
HMIN	Minutes of launch hour
HMU	Earth's gravitation constant
HRS	Hours of launch day
HZERO	Altitude of bottom of ionosphere
H2	Lower limit of ionosphere
H4	Upper limit of ionosphere
IFLAG	Indicator for frequency of printing Kalman Matrices
IO18	Indicator for writing BCD data on L.T. 18

IOBS	Flag for computation of observations
IRDATA	Flag for inclusion of data rejection option
ITER	Number of iterations remaining
ITSAVE	Data to be generated by program at station if available
ITYPE	Signifies type of data to be expected in determination mode
JDRAG	Index used in drag computations
KDATA	Number of observations to be processed
KLM	Indicator for units of position-velocity coordinates
KLM1	Input coordinate type indicator
KLM2	Indicator for precession-nutation
KLM3	Indicator for time corrections
KOMP	Indicator for criterion leading to rectification
KONG	Indicator for frequency of printing
KPRTR	Indicator for writing ephemeris on L.T. 18
KRF	Indicator for refraction computations
KSTA	Station numbers
KTAB	Number of data points processed
KTHC	Number of iterations in Kepler
KWBMU	Subscripts for RCB arrays
KWDDXI	Subscript for array of perturbation accelerations
KWDXI	Subscript for array of perturbation velocities
KWLIN	Number of second order equations to be solved
KWLIND	Dimension of CWLIN array
KWXI	Subscript for array of perturbation positions
LDDXIA, LDDXIZ, LDXIA, LDXIZ, LXI, LXIT	Subscripts used in numerical integration

MBMAX	Number of working bodies
MREF	Number of reference body
MUD	Indicator whose value represents a certain error condition
MWREF	Maximum number of working bodies
NSLTH	Number of equations to be solved
NTYPE	Array of indicators for observation types
NUM	Array of indicators for frequency of processing of observation types
NUMDAT	Number of observation types used in a run
NUMSTA	Number of stations used in a run
NYEAR	Reference year for ephemeris file
NYEARP	Year of launch
ØRM	Magnitude of position vector
ØVB	Array of components of velocity vector
PDOT	Earth rotation rate
PEEK	Test interval for data generation
PERDRG	Drag perturbation
PEROBL	Oblateness perturbation
PERRAP	Radiation pressure perturbation
PI	180° in radians
PMAT	P matrix in Kalman computations
PRE	Precession Matrix
PRNTFE	Print interval in far earth reference
PRNTNE	Print interval in near earth reference
PSI60	Greenwich hour angle of 1960
PSIB	Greenwich hour angle of epoch

PSIDOT	Siderial-to-solar increment in earth rotation rate
PZERO	Reference electron density
QMAT	Q matrix in Kalman computation
RATE	Data sampling interval step
RC	Instantaneous position vector
RCB	Vector for perturbing body to earth
RCIN	Initial velocity vector
RDC	Instantaneous velocity vector
RDCIN	Initial velocity vector
RDI	Velocity vector at last rectification
RDTB	Two-body velocity vector
RECTT	Time of last rectification
RI	Position vector at last rectification
ROTXY	Rotation of X-Y antenna axes clockwise from north
RTAB	Array of drag values
RTB	Two body position vector
RVB	Vector from vehicle to perturbing body
RCMSCX	
RCMSCY	Components of vector from station to vehicle
RCMSCZ	
SAMPLE	Data sampling interval
SAVEP	Original P-matrix estimate
SCALDS	
SCALEA	
SCALED	Scale factors
SCALEV	
SCALVS	

SDDXI	Array of perturbations due to gravitation
SDT	Variance of timing error
SEC	Seconds of launch minute
SGM	Variance of GM
SILT	Sine of station latitude
SINMAT	Inverse of SMAT
SJ2	Variance of second harmonic
SJ3	Variance of third harmonic
SJ4	Variance of fourth harmonic
SMA	Mean Anomaly
SMAT	S matrix of Kalman computations
SMM	Variance of Moon's gravitational attraction
SN1	Variance of ground index of refraction
SN2	Multiplier times $\overline{\epsilon^2}$ in first iteration
SØMG	Small omega
SQTMU	Square root of HMU
SRM	Differential Eccentric Anomaly
STAC	Array of station coordinates
STAHT	Array of station altitudes
STALN	Array of station longitudes
STALT	Array of station latitudes
STANM	Array of station names
S2U S2V S2W	Station position variances

$VL	Variance of uncertainty in velocity of light
T	Current time
TADD	σ of random noise to be added to observation
TBF TBFD TBG TBGD	f and g coefficients of series in Encke computation
TEBAR	Array of variance-covariance matrix for each station
TFLAG	Sampling time reference
TH	Differential eccentric anomaly (estimate)
THC	Differential eccentric anomaly
TI	Time of last rectification
TIN	Initial time
TIRT	Time of last rectification
TK	Time of next observation
TKEP	Time of Kepler reference
TMAX	Maximum time for first time arc
TMAX2	Maximum time for second time arc
TTABLE	Time for ephemeris reference block
TWOPI	360° in radians
TZERO	Initial time in hours
TZHRS	Hours of initial day
UREF	A reference value of semi-major axis used at rectification
WBMU	Array of gravitational constants for reference bodies.

WBNAME	Array of names of reference bodies
XI	Inclination of orbit
YCOM	Array of computed vaiues of observation
YDOT	Velocity of vehicle in special coordinate system for refraction computations
YOBS	Array of observed values
XNUT	Nutation matrix
XNZ	Reference index of refraction

B. List of Diagnostic Indicators

Several of the subroutines in the program perform tests for certain error conditions. If an error is found to have occurred, an integer value is placed in the error flag variable, MUD. A test for non-zero MUD is made in MAIN at each integration step. If the test is passed, the value of MUD is printed out and the case is terminated.

The following is a list of the various MUD values, along with the subroutine in which each is set and the error conditions which caused it:

MUD	WHERE SET	REASON
350	MAIN	DTFE $<$ DTNE
550	RECT	Excessive Change in A
1010	RECT	A = 0
-1020	RECT	Divide Check or Quotient Overflow
3020	KEPLER	Quotient Overflow or Divide Check
3330	RFRCTN	HT $\leq$ HS

MUD	WHERE SET	REASON
3333	RFRCTN	Cos $\theta > 1$
5005	OSCUL	$A = 0$
5010	OSCUL	Quotient Overflow or Divide Check
5020	OSCUL	Quotient Overflow or Divide Check
5030	OSCUL	Quotient Overflow or Divide Check
5040	OSCUL	Quotient Overflow or Divide Check
5060	OSCUL	Quotient Overflow or Divide Check
8050	DERIV	Divide Check
9100	OBLATE	Divide Check or Quotient Overflow
21100	WORKMU	BMU(K) < 0
21200	WORKMU	K = MREF
77775	RECORD	Data out of Sequence
-77777	MAIN	T $>$ TMAX
SL1	ATANS	both arguments 0

X. SUBROUTINE DIRECTORY

NAME	USES COMMON	DESCRIPTION	PAGE #
GENMAT	X	converts topocentric system to rotating geocentric system	X-23
INPUT	X	controls input	X-24
INT	X	controls entries of integration routine	X-25
KEPLER	X	computes two-body coordinates and f functions	X-26
LIBRA	X	computes lunar libration	X-27
MATINV	X	inverts a matrix	X-28
MINVAR	X	performs minimum variance matrix operations	X-29
MNOB	X	computes lunar oblateness and libration effects	X-30
MTML3		multiplies 3 x 3 matrix	X-31
MTML6		multiplies 6 x 6 matrix	X-32
NUTA	X	computes nutation of earth	X-33
OBLATE	X	computes perturbation derivatives due to oblateness	X-34
OBSER	X	computes observations, partials and $\overline{\epsilon^2}$	X-36
OSCUL	X	computes the osculating elements	X-38
PART	X	computes $\Psi(t, t_o)$	X-40
PREC	X	precession	X-41
PRINT	X	controls output	X-42
RAPS		computes perturbation derivatives due to radiation pressure	X-43
RCTTST	X	tests for a rectification	X-44
RECORD	X	reads observation data	X-45
RECT	X	performs a rectification	X-46
REDUCE	X	reduces an angle to less than π	X-47
RFRCTN	X	computes effect of atmospheric refraction on observations	X-48

NAME	USES COMMON	DESCRIPTION	PAGE #
RWDE6F	X	long term numerical integration package	X-49
SMATRX	X	computes point transition matrix	X-51
SOMEGA	X	computes Keplerian element covariance matrix	X-52
STAPOS	X	computes station positions in inertial coordinate	X-58
SUMARY	X	summarize the residuals	X-59
SYMMAT	X	forces symmetry in an n x n matrix	X-60
VECTOR	X	computes multiples of a one dimensional vector	X-61
WORKMU	X	initializes the planets to be considered	X-62
XFORM	X	converts input coordinates to coordinate system of computation	X-64

Identification

ANDRN SHARE Distribution AA-NDRN. A 704 SAP program that was Fortranized by John Mohan of AMA.

Purpose

To generate a sequence of normally distributed random numbers.

Method

Each entrance into ANDRN will yield one value. The value is obtained by first generating a psuedo-random number and then altering it to satisfy certain criteria that are explained in the SHARE write-up. After many entrances to the routine, a sequence of numbers will have been generated that are characterized as normally distributed with the specified mean and standard deviation.

Usage

Calling sequence

ANS = ANDRN (σ, μ, x)

σ : standard deviation of the distribution

μ : statistical mean of the distribution

X : any large octal number. The number should be input to the main program and changed from run to run in order that unique sequences of pseudo-random numbers are generated from run to run.

Identification

ATANS - Fortran function

Purpose

Computes the arctangent of the argument with optional quadrant assignment.

Usage

ANS = ATANS (Y, X, K)

Computes arctangent of Y/X in degrees where Y and X have sign of sine and cosine respectively.

K = 1	$0 \leq$ angle $\leq$	360
K = 0	$-90 \leq$ angle $\leq$	90
K = -1	$-180 \leq$ angle $\leq$	180

Identification

ATMSFR - Fortran subroutine

Purpose

This subroutine sets up an atmospheric model to be used when the inclusion of aerodynamic drag is desired. This is activated when the drag coefficient and the area-mass ratio of the vehicle are given as input. The atmospheric tables are stored in core. They correspond to model #7, contained in Report #25 (Reference 5) of the Smithsonian Astrophysical Observatory, fitting to the ARDC Model Atmosphere of 1956 (Reference 6) at low altitudes. The units for the air density are grams/cm^3 and the height is given in ER from the center of the earth.

Usage

CALL ATMSFR is performed in the initialization section of the main program.

Identification

CROSS - Fortran subroutine

Purpose

Compute C = $\overline{A} \times \overline{B}$

where $\overline{A}$ and $\overline{B}$ are singly subscripted.

Usage

CALL CROSS (A,B,C)

Identification

CWLAR - Fortran subroutine

Purpose

This subroutine has two distinct functions:

1. computes $\xi = R_C - R_{TB}$

$$\dot{\xi} = \dot{R}_C - \dot{R}_{TB}$$

2. computes $R_C = R_{TB} + \xi$

$$\dot{R}_C = \dot{R}_{TB} + \dot{\xi}$$

Its principal use is to update the instantaneous position and velocity vectors after each integration.

Usage

CALL CWLAR

A (-1) in the list generates the first sequence of equations.

A (+1) in the list generates the second sequence of equations.

Identification

DALFA - Fortran subroutine

Purpose

This subroutine converts the variations in the α parameters to equivalent variations in the state vector by the finite rotation method.

Usage

CALL DALFA

Identification

DATE - Fortran Subroutine

Purpose

To compute the various time parameters required for precession and nutation. (See Appendix A)

Usage

CALL DATE (TB, DT, CT, D)

CT - number of Julian Centuries from 0^h Jan 1, 1950 to present.

D - number of days from 0^h Jan 1, 1950 to present

DT = CT - TB = number of Julian Centuries from present to base date

TB = number of Julian centuries from 0^h Jan 1, 1950 to the base date

Identification

DERIV - Fortran subroutine

Purpose

DERIV computes the components of the perturbation forces arising from the sun, moon, and planetary masses, and from earth's oblateness, drag, and radiation pressure. This routine combines then the components of force from all sources into the Encke vector $\ddot{\xi}$.

Method

See equations 2 through 10.

See flow chart following description of the subroutine.

Symbols

CPOS	block of reference body positions produced by LAG
RCB	block of reference body positions as used by DERIV
RVB	block of vehicle positions
CWLIN	block containing ξ , $\dot{\xi}$, and $\ddot{\xi}$
SDDXI	block of components of $\ddot{\xi}$ for each of the perturbing bodies and the Encke term
PEROBL	oblateness components of $\ddot{\xi}$
PERDRG	drag components
PERRAP	radiation pressure (Dummy)

Note that XI, DXI, and DDXI do not appear with these names in the program. They are a part of CWLIN array as specified by the integration subroutine. They may be obtained as follows:

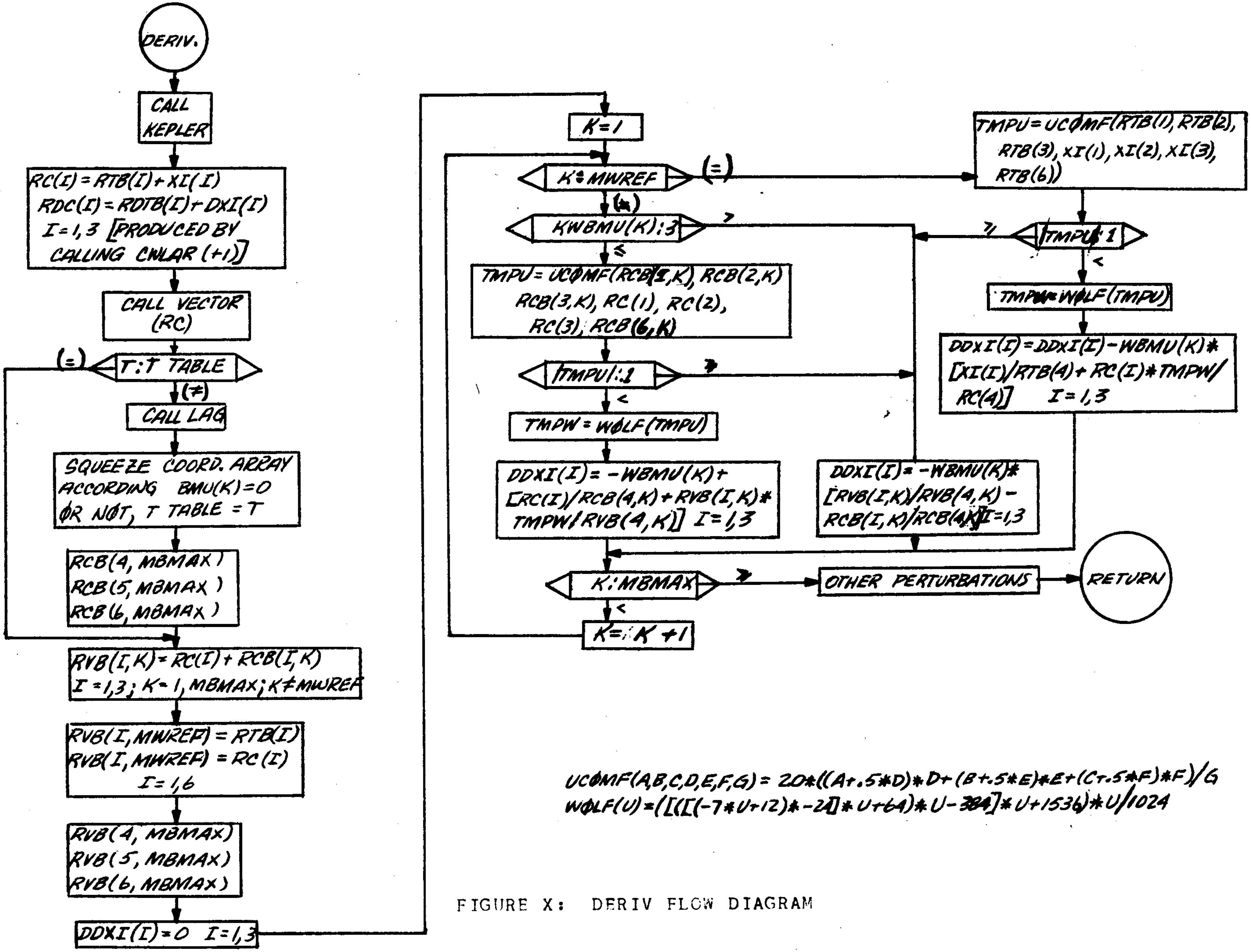
XI(I, J) → CWLIN (1485 - 3 NØSØL + I + 3J)

DXI(I, J) → CWLIN (1485 - 6 NØSØL + I + 3J)

DDXI(I, J) → CWLIN (1485 - 9 NØSØL + I + 3J)

<u>Arrangement of Storage Block for Integration Subroutine</u>

RW DE6F	MINIVAR			
DEQ	KWLIN			
T	T			
DELT	DTI			
XI, DXI, DDXI, CWLIN	CWLIN	XI(3,NØSØL)	→ CWLIN	(1488)
		XI(1,1)	"	(1489-3 NØSØL)
		DXI(3,NØSØL)	"	(1488-3 NØSØL)
		DXI(1,1)	"	(1489-6 NØSØL)
		DDXI(3,NØSØL)	"	(1488-6 NØSØL)
		DDXI(1,1)	"	(1489-9 NØSØL)
		CWLIN etc.,	"	(1488-9 NØSØL)
				(1486-99 NØSØL)



UCØMF(A,B,C,D,E,F,G) = 20*((A+.5*D)*D+(B+.5*E)*E+(C+.5*F)*F)/G

WØLF(U) = ([([(-7*U+12)*-24]*U+64)*U-384]*U+1536)*U/1024

FIGURE X: DERIV FLOW DIAGRAM

Identification

DOT - Fortran function

Purpose

This routine computes the dot product of two vectors.

Usage

ANS = DOT (A,B)

where A and B are the two vectors.

Identification

DRAG - a Fortran routine

Purpose

To compute the perturbations contributed by atmospheric drag.

Method

CALL DRAG

See equation 8. The density is obtained by linear interpolation of density-altitude table.

Symbols

VE	$\dot{R} - \Omega \times R$
FRHOA FRHOB	tables of $-\frac{1}{2}\rho \frac{A}{m} C_D$ versus altitude
DFRHOA DFRHOB	divided differences of the tables
POLAT	interpolated value of $-\frac{1}{2}\rho \frac{A}{m} C_D$ multiplied by $\left\lvert \dot{R} - \Omega \times R \right\rvert$
PERDAG	x,y,z components of drag acceleration

Identification

EOFIX -- FAP subroutine with two entries.

Purpose

This subroutine avoids termination by EXEM upon encountering an END OF FILE by adjusting EXEM.

Usage

```
CALL EOFIX (IND)
IF (IND) 1,2,1
1  ERROR RETURN; EOF ENCOUNTERED
2  READ TAPE NO., LIST
```

CALL EXEMR to reset EXEM.

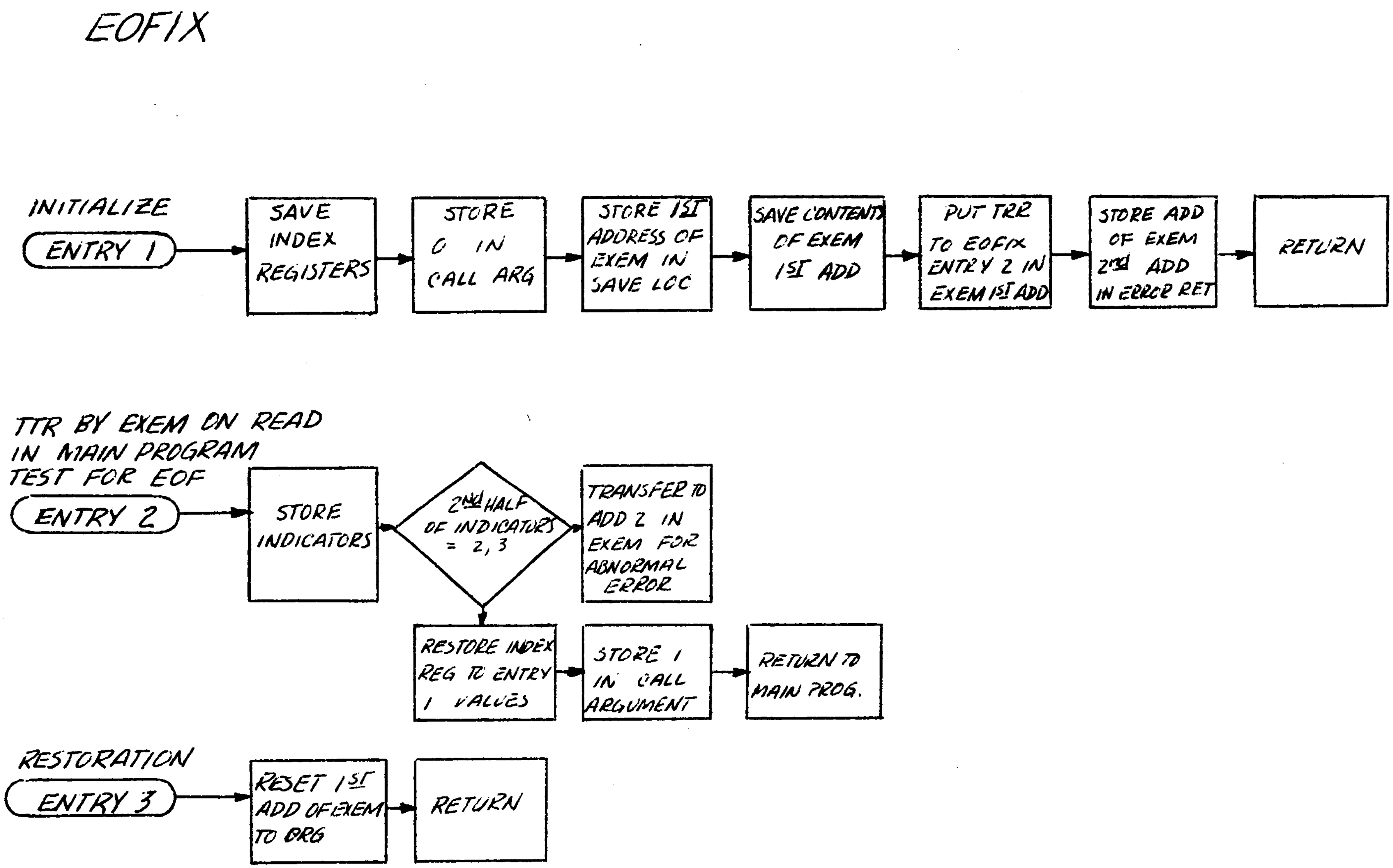


FIGURE XI: CEFIX FLOW DIAGRAM

Identification

EPHEM — A FAP subroutine with three entries.

1. READ1 Ephemeris tape read and core set up routine
2. READ2 Not currently used
3. LAG Interpolation for position and maintenance of current planetary positions
4. REFSWT Reference ephemeris data to specified body.

Purpose

To set up in core a table of planetary positions referenced to a specifiable body, to read in additional data when necessary, and to interpolate in the table for position. See Appendix F for a description of the treatment of Planetary Coordinates.

Usage

1. READ1

   Calling sequence (Fortran)

   CALL READ1

   The following common storage must be set up:

   NYEAR : Year of launch

   TZERO : Time in hours from base time*

   DAYS : Launch time, days

   HRS : Launch time, hours

   MIN : Launch time, minutes

   SEC : Launch time, seconds

   MREF : Reference body

*Base time is 0.0 hours U.T. of December 31 of the year previous to launch.

Usage (continued)

LAG

Calling sequence (Fortran)

Call LAG

The following common storage must be set up

T = Time, measured in hours from launch

READ1 must be called once to set tables in core. LAG will keep tables up to date.

REFSWT

Calling sequence (Fortran)

Call REFSWT

The following common storage must be set up

MREF - Reference body

READ1 gives position vectors with respect to the reference body indicated by MREF

MREF = 1	Earth reference
MREF = 2	Sun reference
MREF = 3	Moon reference
MREF = 4	Venus reference
MREF = 5	Mars reference

To change the reference body, change MREF and CALL REFSWT

Identification

EXPR - Fortran subroutine

Purpose

To compute common terms for precession and nutation. (See Appendix A)

Usage

CALL EXPR (T, D, SPI, EQ, E, XØ, XC)

T   time in Julian centuries

D   Julian days from present to 0^h Jan 1 1950

E   nutation in obliquity

EQ   mean obliquity

PSI nutation in longitude

XØ   mean longitude of the moon

XC   mean longitude of the descending node of the moon's mean equator

Identification

FIX - Fortran subroutine

Purpose

To unpack observation data type designation.

Usage

Call FIX

OCT: An actual number whose bits represent whether an observation type is available or not. A zero (0) means it is not, a one (1) that it is. Proceeding from right to left the bits represent azimuth, elevation, range, range rate, right ascension, declination, L, M, X, Y, Δt, $\Delta t'$, the rest are open.

The program unpacks this number and stores each bit in the decrement of NTYPE which is dimensioned sixteen (16)

NTYPE (1) corresponds to azimuth

NTYPE (2) corresponds to elevation

NTYPE (16) corresponds to open

NUMDAT: Represents the total number of observables available at any time point from one station.

Identification

GAMMAT - Fortran Subroutine

Purpose

GAMMAT computes the rotation matrix that transforms a vector from a rotating geocentric system to the inertial system.

Method

See Appendix A

Usage

CALL GAMMAT (CONST, GAM, GAMS, DI)

CONST	earth's rotation rate
GAM	transformation matrix
GAMS	Greenwich Hour Angle
DI	integer number of days since 0^h Jan 1, 1950

Identification

GENMAT - Fortran Subroutine

Purpose

GENMAT is a general purpose orthogonal transformation matrix.

Method

See Appendix A

Usage

CALL GENMAT (XLAT, XLONG, GHA)

XLAT = latitude

XLONG = longitude

GHA = Greenwich hour angle

Identification

INPUT — a Fortran subroutine

Purpose

This routine controls the input to the program. The input is divided into eleven sections. When running consecutive cases, only the section in which the input has changed need be input. This routine will also prepare the observation data tape if the observations are input on cards.

Usage

CALL INPUT

Identification

INT → A Fortran subroutine.

Purpose

The routine was designed to make it easier to exchange the present integration package, RWDE6F, for another one. It has four distinct entries:

1. Initialization
2. Normal backward difference entry
3. Runge-Kutta entry
4. Change integration interval

Usage

The entries are achieved by using a 0, 1, 2, or 3 in the calling sequence. The subroutine RWDE6F should be read for a more detailed explanation of the linkage.

Identification

KEPLER - a Fortran routine

Purpose

To solve the Herrick's two-body equations for a time greater or less than rectification time.

Method

See section IV.

Usage

CALL KEPLER

Identification

LIBRA — Fortran subroutine

Purpose

To transform from the moon fixed axes $\hat{X}_M$ $\hat{Y}_M$ $\hat{Z}_M$ to the $\hat{X}_E$ $\hat{Y}_E$ $\hat{Z}_E$ axes.

Method

See Appendix A

Usage

CALL LIBRA

Identification

MATINV - Share distribution AN-F402, a 704 Fortran program.

Purpose

Solution of the inverse of a non-singular matrix.

Method

Gauss-Jordan elimination method is used to invert the matrix.

Usage

CALL MATINV (A,N,B,M, DETERM)

A: Matrix to be inverted

N: Order of the matrix A

B: Not defined for inverse solution but storage must be allocated

M: A zero denotes that MATINV is to be used only for the inversion of the matrix A.

DETERM: determinant of the matrix a.

The inverse appears in A after return to calling program.

Identification

MINVAR — Fortran subroutine

Purpose

A variable order matrix multiplication routine specifically designed for the minimum variance matrix calculation. The subscript N in the description below is the quantity that may vary. It refers to the number of pieces of data that you wish to process through the minimum variance filter.

Usage

CALL MINVAR

Enter with M_{Nx6}, S_{6x6}, $\bar{Q}_{6x6}$ and $\overline{\epsilon^2}_{NxN}$

Exit with Q^+_{6x6}, P^+_{6x6}, and L_{6xN}

Subroutine computes

$$N_{Nx6} = M_{Nx6} \; S_{6x6}$$

$$A_{6xN} = Q^-_{6x6} \; N^*_{6xN}$$

$$Y_{NxN} = N_{Nx6} \; A_{6xN} + \overline{\epsilon^2}_{NxN}$$

$$L_{6xN} = A_{6xN} \; Y^{-1}_{NxN}$$

$$Q^+_{6x6} = Q^-_{6x6} - L_{6xN} \; N_{Nx6} \; Q^-_{6x6}$$

$$P^+_{6x6} = S_{6x6} \; Q^+_{6x6} \; S^*_{6x6}$$

Identification

MNOB — Fortran subroutine

Purpose

To compute the effect of lunar oblateness and libration on the gravitational field.

Method

See Appendix A, OBLATE and LIBRA.

Usage

CALL MNOB (PERMN)

PERMN = perturbation due to moon's oblateness.

Identification

MTML3 — Fortran subroutine

Purpose

To compute the product of two 3 x 3 matrices.

Usage

CALL MTML3 (A,B,C,K)

If K = 1

C = A x B

If K = 0

C = A x B*

Identification

MTML6 - Fortran subroutine

Purpose

To compute the product of two 6 x 6 matrices.

Usage

CALL MTML6 (A,B,C,K)

If K = 1

$C = A \times B$

If K = 0

$C = A \times B^*$

Identification

NUTA — Fortran subroutine

Purpose

To compute the coordinate transformations describing the earth's nutation.

Method

See Appendix A

Usage

CALL NUTA

Identification

OBLATE - a Fortran subroutine

Purpose

To compute the perturbations contributed by the non-spherical shape of the earth, and the effect of precession and nutation.

Method

See equations (6) and (7) of Section IV and Appendix A.

The earth oblateness potential may be written as

$$\phi = \frac{-\mu}{r}\left\{\frac{J_{20}}{r^2}\left[\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right] + \frac{J_{30}}{r^3}\left[\frac{5}{2}\left(\frac{z}{r}\right)^3 - \left(\frac{3}{2}\right)\left(\frac{z}{r}\right)\right] + \frac{J_{40}}{r^4}\left[\frac{35}{8}\left(\frac{z}{r}\right)^4 - \frac{15}{4}\left(\frac{z}{r}\right)^2 + \frac{3}{8}\right]\right\}$$

where the equatorial radius of the earth is the unit of length and where the coefficients J_{20}, J_{30}, J_{40} are assigned to the following numerical values (given by equations (8) and (13) of reference 7):

$J_{20} = 1082.3 \times 10^{-6}$

$J_{30} = -2.3 \times 10^{-6}$

$J_{40} \doteq -1.8 \times 10^{-6}$

The perturbation accelerations due to the earth's oblateness are given by equations (6) and (7) of this document. As actually programmed in the minimum variance program, the functions b and c of equation (7) appear as

$$b = \frac{CONJ}{r^5}\left[\frac{5}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right] + \frac{CONA}{r^6}\left[\frac{15}{2}\left(\frac{z}{r}\right) - \frac{35}{2}\left(\frac{z}{r}\right)^3\right]$$

$$+ \frac{CONK}{r^7}\left[-63\left(\frac{z}{r}\right)^4 + 42\left(\frac{z}{r}\right)^2 - 3\right]$$

$$C = \frac{CONJ}{r^5}\left[-z\right] + \frac{CONA}{r^5}\left[\frac{15}{2}\left(\frac{z}{r}\right)^2 - \frac{3}{2}\right] + \frac{CONK}{r^6}\left[-12\left(\frac{z}{r}\right) + 28\left(\frac{z}{r}\right)^3\right]$$

where

CONJ = $3\mu J_{20}$ = $2\mu J_2$ of equation 7 of Section IV

CONA = $-\mu J_{30}$ = μJ_3 of equation 7 of Section IV

CONK = $-5/8\mu J_{40}$ = $5\mu J_4$ of equation 7 of Section IV

Note that the corresponding input quantities to the program are

CONJR = $3/2\mu J_{20}$ = μJ_2 of equation 7 of Section IV

CONAR = $-\mu J_{30}$ = μJ_3 of equation 7 of Section IV

CONKR = $-15/4\mu J_{40}$ = $30\mu J_4$ of equation 7 of Section IV

Identification

OBSER — a Fortran subroutine

Purpose

This routine computes the observables for a particular station and the residuals which consist of the differences between the computed values of the observables and the observation data. It also computes the partial derivatives of the observables with respect to the state variables. It is capable of this for azimuth, elevation, range, range rate, right ascension and declination, l direction cosine and m direction cosine, X and Y angles, and Δt and $\Delta t'$ (range and range rate equivalents). It also computes the time variation of $\overline{\epsilon^2}$ as a function of a variety of errors, such as station location, timing errors, etc.

If the program is operating in the data generating (reference) mode it will generate a binary tape containing the observation, the time of the observations, the type of observations made and the station from which the observations were made. This tape may then be used as a simulation of real data for the orbit determination mode of the program. If the program is operating in the determination mode it will generate another tape that contains the observations, the residuals and the data type so that a summary of this information may be made at the completion of the case.

Usage

CALL OBSER

Method

See equations (47) through (82).

Identification

OSCUL - Fortran Subroutine

Purpose

The purpose of this routine is to compute the classical osculating Keplerian elements.

Method

The osculating elements are obtained from the following equations:

$$a = \left(\frac{2}{r} - \frac{V^2}{\mu}\right)^{-1}$$

$$n = \mu^{1/2} |a|^{-3/2}$$

$$\left.\begin{array}{l} e \cos E \\ e \cosh E \end{array}\right\} = 1 - \frac{r}{a} \qquad M = \left\{\begin{array}{l} E - e \sin E \\ e \sinh E - E \end{array}\right.$$

$$\left.\begin{array}{l} e \sin E \\ e \sinh E \end{array}\right\} = \frac{d}{\sqrt{|\mu a|}} \qquad t_{pc} = t - \frac{M}{n}$$

The angles Ω, ω, i are obtained from the vectors H and P, where

$$H = R \times \dot{R}$$

$$e\hat{P} = \left(\frac{1}{r} - \frac{1}{a}\right) R - \frac{d}{\mu} \dot{R}$$

In terms of these vectors:

$$\cos i = \frac{H_z}{h}$$

$$\sin \Omega = \frac{H_x}{h \sin i}$$

i in the first or fourth quadrant

$$\cos \Omega = \frac{-H_y}{h \sin i}$$

$$\cos \omega = P_x \cos \Omega + P_y \sin \Omega$$

$$\sin \omega = \frac{P_z}{\sin i}$$

<u>Usage</u>

CALL OSCUL

Identification

PART - Fortran Subroutine

Purpose

This subroutine computes the parameter transition matrix $\psi(t,t_r)$ at any time. It must be called at rectification. If the rectification is due to a minimum variance correction, it sets $\psi = I$.

Usage

CALL PART

The parameter transition matrix, ψ is given by equation (30).

Identification

PREC - Fortran subroutine

Purpose

To compute the coordinate transformations due to the earth's precession.

Method

See Appendix A

Usage

```
CALL PREC
```

Identification

PRINT -- Fortran subroutine

Purpose

This subroutine controls the output concerning the trajectory of the vehicle. It will output R_{VC} and $\dot{R}_{VC}$ in E.R. and E.R./hr or KM and KM/hr depending upon the input option KLM. The time at which output is given is determined by PRNTDT, a print frequency that is a function of the input parameters PRNTNE and PRNTFE. This routine also controls the output of the classical astronomical elements.

Usage

CALL PRINT

Identification

RAPS - Fortran subroutine

Purpose

A dummy subroutine that is intended to compute the perturbation accelerations due to solar radiation pressure.

Usage

CALL RAPS

Identification

RCTTST - A Fortran subroutine.

Purpose

The two-body orbit is rectified whenever the perturbations exceed specified maximum values. The following three tests are made:

1. $\dfrac{|\xi|}{|R_c|} > .02$

2. $\dfrac{|\dot{\xi}|}{|R_c|} > .02$

3. $u = \dfrac{2}{r_{TB}^2}\left[R_{TB} + \dfrac{\xi}{2}\right] \cdot \xi > .05$

Usage

CALL RCTTST

The value KOMP, which is in COMMON, will contain an integer which corresponds to the test that was failed upon return to calling program. Otherwise KOMP = 0.

Identification

RECORD - Fortran subroutine

Purpose

The reading of the observation data tape is performed by this routine. It is read so that there are always two records in core at any one time. A record consists of a time, observation data and the associated station and the observation data type. If there is more than one record at the same time due to simultaneous observations from different stations, the routine will read ahead to see how many records have the same time. It will return to the main program with this count in COMMON.

This routine also corrects the time recorded in the raw data to cause it to correspond to the time the signal was transmitted from the vehicle. It also resolves ambiguities in the Goddard R & $\dot{R}$ system data.

Method

The time correction technique is discussed in Section VI-C. The ambiguity resolution problem is discussed in Appendix D.

Usage

CALL RECORD

Identification

RECT - Fortran subroutine

Purpose

This subroutine computes the parameters pertinent to a rectification. The three basic terms that are computed are a, n and $R_r \cdot \dot{R}_r$ where the subscript r refers to the value R_c and $\dot{R}_c$ at the time of rectification.

Usage

CALL RECT

Identification

REDUCE - Fortran function

Purpose

Reduces an angle to less than π

Usage

ANGLE = REDUCE (ANGLE)

Identification

RFRCTN - Fortran subroutine

Purpose

Computes corrections to all observables due to propagation of electromagnetic waves through a refractive medium.

Method

See Appendix C

Usage

CALL RFRCTN

Identification

RWDE6F - Share Distribution No. 775. A 704 SAP program that was Fortranized by Leon Lefton of AMA.

Purpose

To solve a set of N simultaneous second order differential equations.

Method

A fourth-order Runge-Kutta method is used to start the integration and to change the step-size during integration. A Cowell "second-sum" method based on sixth order differences is used to continue the integration. Double precision is used internally to control round-off errors. Truncation error can be controlled by choosing an appropriate step-size. The values of the variables and derivatives are consistent at all times.

Usage

1) For initialization

CALL DEIN (NEQ, DERIV, IORD, K, EPS, HMIN, HMAX, YMIN, DPT, ACC)

NEQ: Number of differential equations

DERIV: Name of derivative routine

IORD: Order of backward difference scheme

K : Ratio of Cowell step size to Runge-Kutta step size

EPS : Convergence criterion

HMIN : Minimum step size

HMAX : Maximum step size

YMIN : Minimum function value allowed

DPT : Least significant part of time

ACC : Routine places + value when in Runge-Kutta and - value when in Cowell.

2) Normal entry

CALL DEREG

3) Change integration interval

CALL DECHA (Nt)

4) Runge-Kutta integration with specified Δ t, CALL DERKI

The location of NEQ is considered as the address that begins the block of information concerning RWDE6F. Following NEQ there is

time

Δ t

y_1 - N

$\dot{y}_1$ - N

$\ddot{y}_1$ - N

The share write up should be read for a detailed description of RWDE6F.

Identification

SMATRX – FORTRAN subroutine

Purpose

This subroutine computes the point transformation matrix S or its inverse.

Usage

CALL SMATRX

A +1 in the list will generate S

A -1 in the list will generate S^{-1}

The S matrix and its inverse are given in equations (23) and (25).

Identification

SOMEGA - a Fortran subroutine

Purpose

To provide standard deviations in the estimates of the Keplèrian orbital elements.

Method

1. Obtain the S matrix which is the partial derivatives of the state variables $x \rightarrow \dot{z}$ with respect to the Keplerian orbital elements

   ω - argument of perigee

   Ω - right ascension of ascending node

   i - inclination

   e - eccentricity

   M - mean anomaly

   n - mean motion

2. Find the required covariance matrix from

$$Q_{KEP} = \left(S_{KEP}^{-1}\right) P \left(S_{KEP}^{-1}\right)^{*}$$

P is the state vector covariance matrix

3. Equations programmed are as follows:

$$e = \left[\frac{\bar{r}\cdot\dot{\bar{r}}}{\mu a} + \left(1 - \frac{r}{a}\right)^{2}\right]^{1/2}$$

If e > 0 and a > 0 then

$$SIN\ E = \frac{\bar{r}\cdot\dot{\bar{r}}}{\sqrt{\mu a}\ e}$$

$$\cos E = \frac{(1 - \frac{r}{a})}{e}$$

$$P_x = \frac{\cos E}{r} x - \sin E \sqrt{\frac{a}{\mu}} \dot{x}$$

$$P_y = \frac{\cos E}{r} y - \sin E \sqrt{\frac{a}{\mu}} \dot{y}$$

$$P_z = \frac{\cos E}{r} z - \sin E \sqrt{\frac{a}{\mu}} \dot{z}$$

If $e \geqq 0$ then

$$H_x = y\dot{z} - z\dot{y}$$

$$H_y = z\dot{x} - x\dot{z}$$

$$H_z = x\dot{y} - y\dot{x}$$

$$\cos i = H_z / (H_x^2 + H_y^2 + H_z^2)^{1/2}$$

$$\sin i = (H_x^2 + H_y^2)^{1/2} / (H_x^2 + H_y^2 + H_z^2)^{1/2}$$

If $\sin i \neq 0$ then

$$\sin \Omega = H_x / (H_x^2 + H_y^2)^{1/2}$$

$$\cos \Omega = -H_y / (H_x^2 + H_y^2)^{1/2}$$

If $\sin i \neq 0$ and $e > 0$ then

$$\sin \omega = \frac{P_z}{\sin i}$$

$$\cos \omega = P_x \cos \Omega + P_y \sin \Omega$$

$$\omega = \tan^{-1}\left(\frac{\sin \omega}{\cos \omega}\right)$$

If sin i = 0 and e = 0 then

$\cos \Omega = 1$

$\sin \Omega = 0$

Regardless of sin i, If e = 0 then

$\omega = 0$

$\cos \omega = 1$

$\sin \omega = 0$

$\sin E = 0$

$\cos E = 1$

If sin i = 0 and e > 0 then

$\sin \Omega = 0$

$\cos \Omega = 1$

$\sin \omega = P_y$

$\cos \omega = P_x$

$\omega = \tan^{-1}\left(\frac{\sin \omega}{\cos \omega}\right)$

The S matrix is programmed as follows

$$S(1,1) = \frac{\partial x}{\partial \omega} = -r(\sin l \cos \Omega + \cos l \cos i \sin \Omega)$$

$$S(2,1) = \frac{\partial y}{\partial \omega} = -r(\sin l \sin \Omega - \cos l \cos i \cos \Omega)$$

$$S(3,1) = \frac{\partial z}{\partial \omega} = r \cos l \sin i$$

where $l = v + \omega$

$v = \tan^{-1}\left(\frac{\sqrt{1-e^2}\sin E}{\cos E - e}\right)$

$$S(4,1) = \frac{\partial \dot{x}}{\partial \omega} = -\dot{Y}_N \cos\Omega - \dot{X}_N \cos i \sin\Omega$$

$$S(5,1) = \frac{\partial \dot{y}}{\partial \omega} = -\dot{Y}_N \sin\Omega + \dot{X}_N \cos i \cos\Omega$$

$$S(6,1) = \frac{\partial \dot{z}}{\partial \omega} = \dot{X}_N \sin i$$

where

$$\dot{X}_N = \sqrt{\frac{\mu}{p}}\left(\sin l + e \sin\omega\right)$$

$$\dot{Y}_N = \sqrt{\frac{\mu}{p}}\left(\cos l + e \cos\omega\right)$$

$$p = a(1-e^2)$$

$$S(1,2) = \frac{\partial x}{\partial \Omega} = -y$$

$$S(2,2) = \frac{\partial y}{\partial \Omega} = x$$

$$S(3,2) = \frac{\partial z}{\partial \Omega} = 0$$

$$S(4,2) = \frac{\partial \dot{x}}{\partial \Omega} = -\dot{y}$$

$$S(5,2) = \frac{\partial \dot{y}}{\partial \Omega} = \dot{x}$$

$$S(6,2) = \frac{\partial \dot{z}}{\partial \Omega} = 0$$

$$S(1,3) = \frac{\partial x}{\partial i} = z \sin \Omega$$

$$S(2,3) = \frac{\partial y}{\partial i} = -z \cos \Omega$$

$$S(3,3) = \frac{\partial z}{\partial i} = r \sin \ell \cos i$$

$$S(4,3) = \frac{\partial \dot{x}}{\partial i} = \dot{z} \sin \Omega$$

$$S(5,3) = \frac{\partial \dot{y}}{\partial i} = -\dot{z} \cos \Omega$$

$$S(6,3) = \frac{\partial \dot{z}}{\partial i} = \dot{Y}_N \cos i$$

$$S(1,4) = \frac{\partial x}{\partial e} = -\frac{x}{r} a \cos v + \frac{\partial x}{\partial \omega} \frac{\partial \ell}{\partial e}$$

$$S(2,4) = \frac{\partial y}{\partial e} = -\frac{y}{r} a \cos v + \frac{\partial y}{\partial \omega} \frac{\partial \ell}{\partial e}$$

$$S(3,4) = \frac{\partial z}{\partial e} = -\frac{z}{r} a \cos v + \frac{\partial z}{\partial \omega} \frac{\partial \ell}{\partial e}$$

where

$$\frac{\partial \ell}{\partial e} = \sin v \left(\frac{a}{r} + \frac{1}{1-e^2} \right)$$

$$S(4,4) = \frac{\partial \dot{x}}{\partial e} = \frac{ae}{p} \dot{x} - \frac{x}{r}\sqrt{\frac{\mu}{p}} \frac{\partial \ell}{\partial e} - \sqrt{\frac{\mu}{p}} \left[\cos \Omega \sin \omega + \cos i \sin \Omega \cos \omega \right]$$

$$S(5,4) = \frac{\partial \dot{y}}{\partial e} = \frac{ae}{p} \dot{y} - \frac{y}{r}\sqrt{\frac{\mu}{p}} \frac{\partial \ell}{\partial e} - \sqrt{\frac{\mu}{p}} \left[\sin \Omega \sin \omega - \cos i \cos \Omega \cos \omega \right]$$

$$S(6,4) = \frac{\partial \dot{z}}{\partial e} = \frac{ae}{p}\dot{z} - \frac{z}{r}\sqrt{\frac{\mu}{p}}\,\frac{\partial \ell}{\partial e} + \sqrt{\frac{\mu}{p}}\left[\cos\omega \,\sin i\right]$$

$$S(1,5) = \frac{\partial x}{\partial M} = \frac{ae \sin v}{\sqrt{1-e^2}}\,\frac{x}{r} + \frac{a^2\sqrt{1-e^2}}{r^2}\,\frac{\partial x}{\partial \omega}$$

$$S(2,5) = \frac{\partial y}{\partial M} = \frac{ae \sin v}{\sqrt{1-e^2}}\,\frac{y}{r} + \frac{a^2\sqrt{1-e^2}}{r^2}\,\frac{\partial y}{\partial \omega}$$

$$S(3,5) = \frac{\partial z}{\partial M} = \frac{ae \sin v}{\sqrt{1-e^2}}\,\frac{z}{r} + \frac{a^2\sqrt{1-e^2}}{r^2}\,\frac{\partial z}{\partial \omega}$$

$$S(4,5) = \frac{\partial \dot{x}}{\partial M} = -\frac{a^2\sqrt{1-e^2}}{r^2}\sqrt{\frac{\mu}{p}}\,\frac{x}{r}$$

$$S(5,5) = \frac{\partial \dot{y}}{\partial M} = -\frac{a^2\sqrt{1-e^2}}{r^2}\sqrt{\frac{\mu}{p}}\,\frac{y}{r}$$

$$S(6,5) = \frac{\partial \dot{z}}{\partial M} = -\frac{a^2\sqrt{1-e^2}}{r^2}\sqrt{\frac{\mu}{p}}\,\frac{z}{r}$$

$$S(1,6) = \frac{\partial x}{\partial n} = -\frac{2x}{3n}$$

$$S(2,6) = \frac{\partial y}{\partial n} = -\frac{2y}{3n}$$

$$S(3,6) = \frac{\partial z}{\partial n} = -\frac{2z}{3n}$$

$$S(4,6) = \frac{\partial \dot{x}}{\partial n} = \frac{\dot{x}}{3n}$$

$$S(5,6) = \frac{\partial \dot{y}}{\partial n} = \frac{\dot{y}}{3n}$$

$$S(6,6) = \frac{\partial \dot{z}}{\partial n} = \frac{\dot{z}}{3n}$$

Identification

STAPOS — Fortran subroutine

Purpose

To compute the current station location in inertial coordinates. See Equation 47.

Usage

CALL STAPOS

Identification

SUMARY — Fortran subroutine

Purpose

This routine summarizes the residuals in observations and/or the state variables. The residuals in the observations are output next to the observations themselves. The time, in hours, min., and seconds, from epoch is also given. The residuals in the state variables may also be obtained if a tape of the reference orbit ephemeris from which the observations were generated is supplied on L.T.-19. The time, as defined above, is given. The component errors, magnitude error and also the relative error is given for the position and velocity vectors. The relative error is the magnitude of the error vector divided by the magnitude of the vector. All results are labeled by name and dimension.

Usage

CALL SUMARY

Identification

SYMMAT — Fortran subroutine

Purpose

To force symmetry in n x n matrices.

Method

$$A_{ij} = \frac{A_{ij} + A_{ji}}{2}$$

Usage

CALL SYMMAT (A)

A = matrix name

Identification

VECTOR — Fortran subroutine

Purpose

The routine computes the 4th, 5th and 6th terms of a one dimensional vector.

4th term $= |V|^3$

5th term $= |V|$

6th term $= |V|^2$

Usage

```
CALL VECTOR (V)
```

V is the vector of interest.

Identification

WORKMU - Fortran subroutine

Purpose

The program is capable of including the gravitational attraction of six perturbing bodies.

1. Earth
2. Sun
3. Moon
4. Venus
5. Mars
6. Jupiter

If any of these bodies are not desired, (see input section #2, card #3), this subroutine will pack the various arrays that are associated with these bodies to simplify the logic in generating the gravitational perturbation.

Usage

CALL WORKMU is performed in the initialization section of the main program.

Symbols

HMU	μ of reference body
BMU	table of 6 mass parameters
WMBU	table of working mass parameters
BNAME	names of 6 objects
MBMAX	number of working objects
KWBMU	table relating indexing of working objects to the original 6

Identification

XFORM — FORTRAN subroutine

Purpose

To convert various input position and velocity representations to Cartesian inertial position and velocities. See Appendix A.

Usage

CALL XFORM

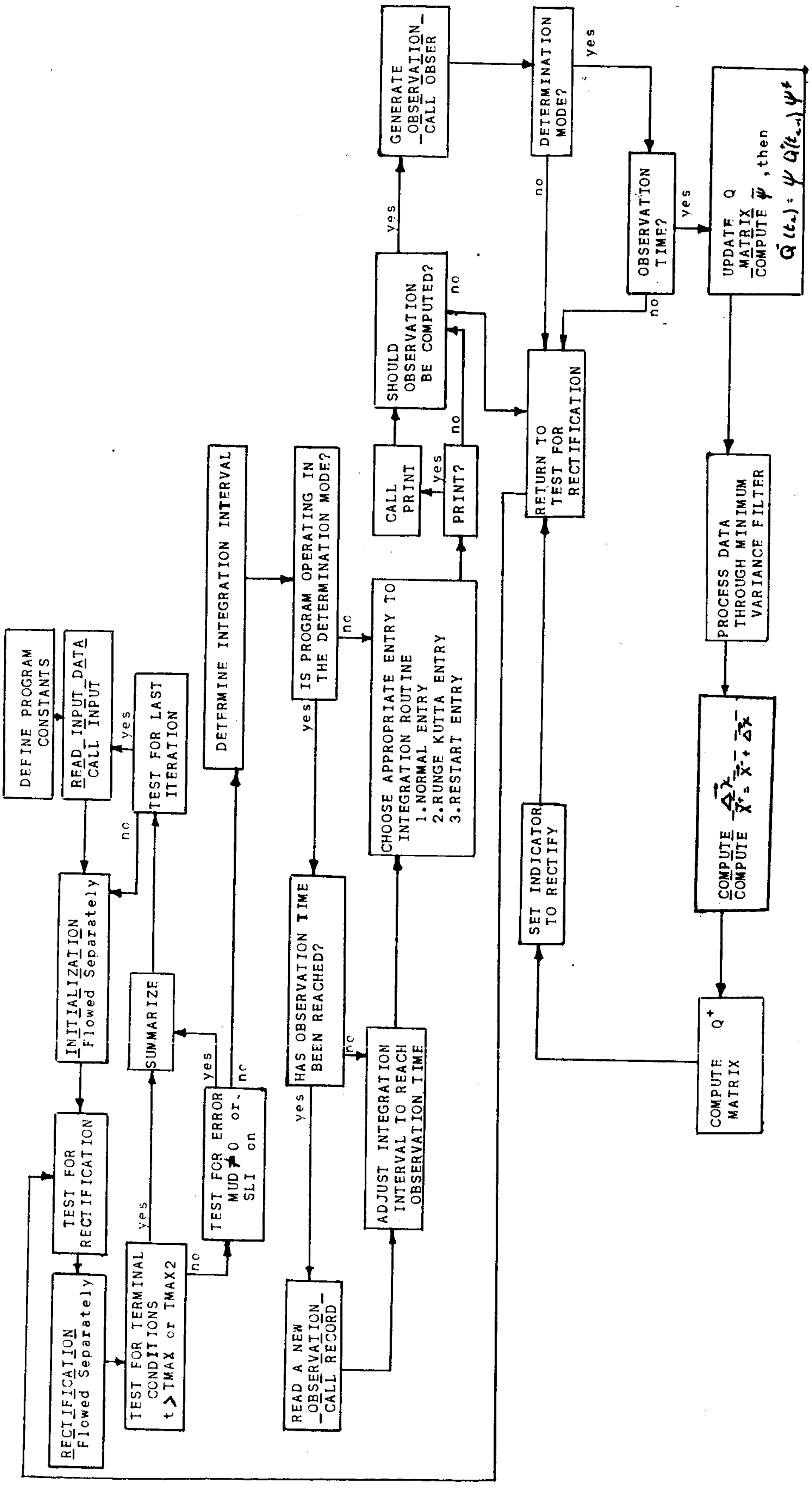


FIGURE XII
MAIN PROGRAM FLOW

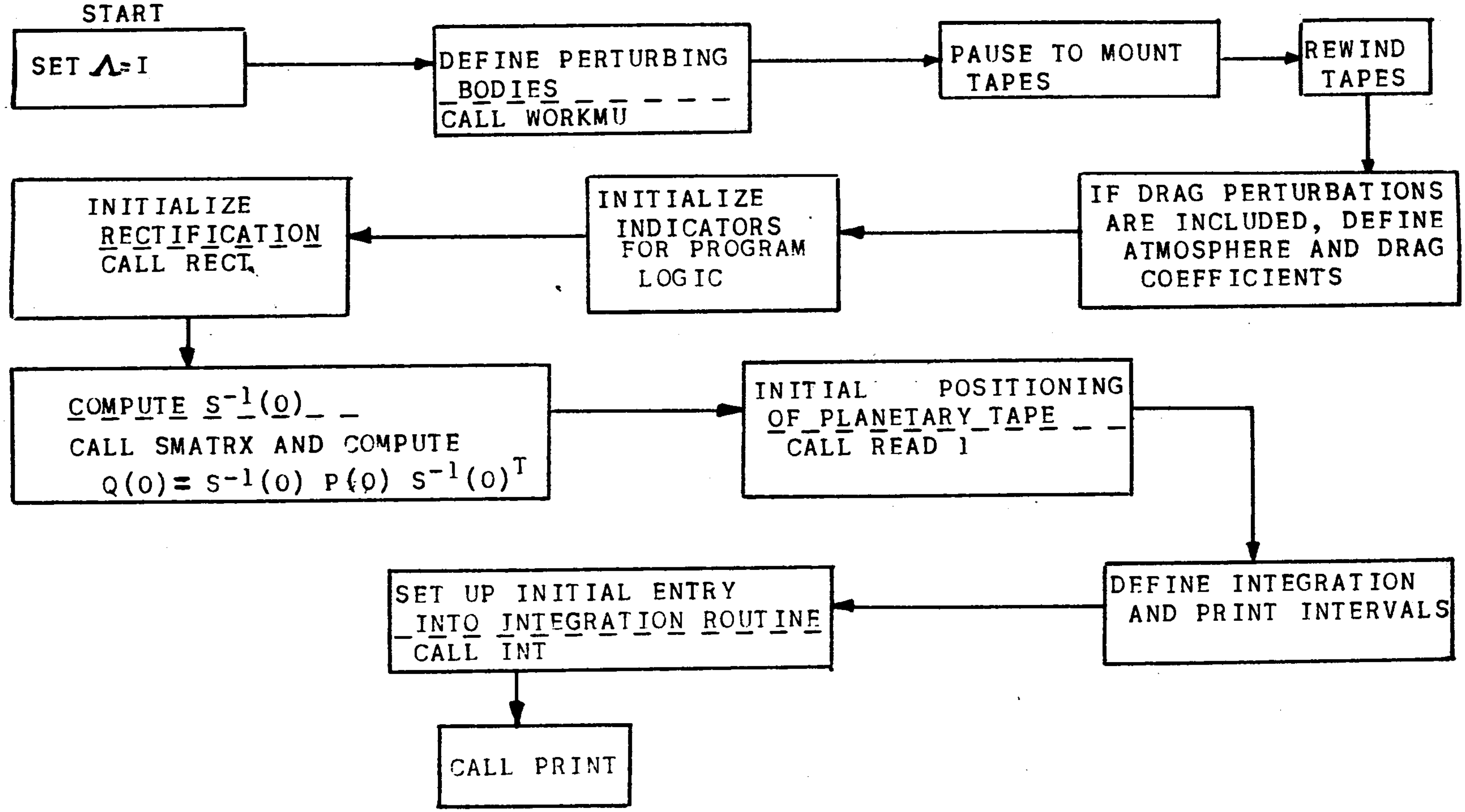


FIGURE XIII: INITIALIZATION SECTION OF MAIN PROGRAM

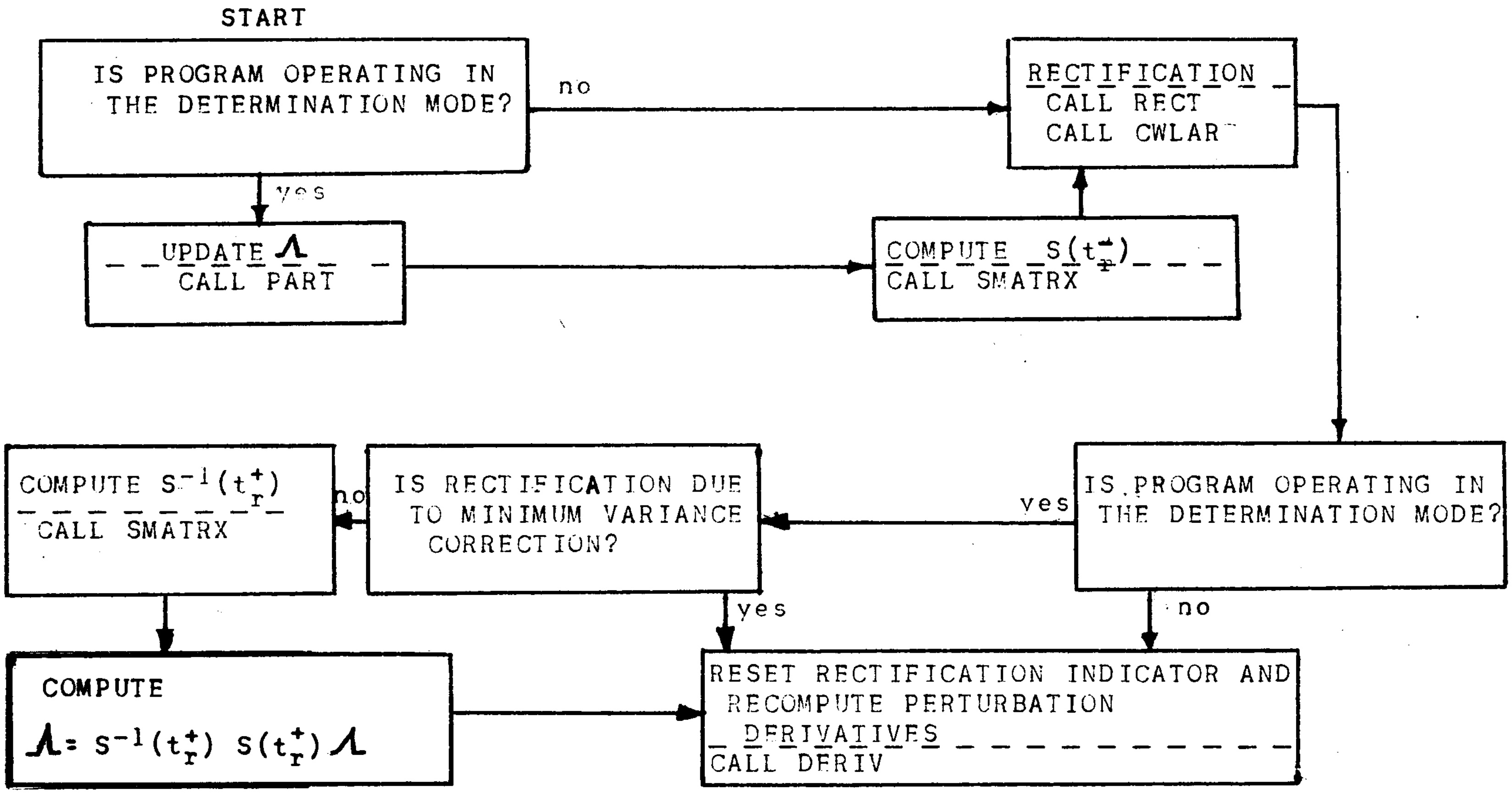


FIGURE XIV: RECTIFICATION SECTION OF MAIN PROGRAM

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Appendices

APPENDIX A

DESCRIPTION OF SUBROUTINES USED IN TRANSFORMATIONS

1.0 Subroutines for Transformations

1.1 PREC: Precession

The spin axis of the earth is slowly precessing in inertial space because of the lunar and solar attractions on the terrestrial bulge; the plane of the earth's orbit about the sun (ecliptic) moves slowly because of planetary attractions. As a result the intersection of the earth's mean equator and the ecliptic (termed the first point in Aries ♈) undergoes a gradual rotation in space. Therefore, the $\hat{X}_Q, \hat{Y}_Q, \hat{Z}_Q$ coordinate system as defined in Section VI is rotating with respect to the $\hat{X}_B, \hat{Y}_B, \hat{Z}_B$ system. Figure A-1 illustrates this rotation of the equinox with respect to its position as base date. The transformation from $\hat{X}_Q, \hat{Y}_Q, \hat{Z}_Q$ to $\hat{X}_B, \hat{Y}_B, \hat{Z}_B$ is $[A]$:

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} \Delta a_{11} & \Delta a_{12} & \Delta a_{13} \\ \Delta a_{21} & \Delta a_{22} & \Delta a_{23} \\ \Delta a_{31} & \Delta a_{32} & \Delta a_{33} \end{bmatrix}$$

$$a_{11} = 1.0000000 - 0.000296970\, T_B^2 - 0.000000130\, T_B^3$$

$$a_{12} = 0.02234988\, T_B + 0.00076700\, T_B^2 - 0.00000221\, T_B^3$$

$$a_{13} = 0.00971711\, T_B - 0.00000207\, T_B^2 - 0.00000096\, T_B^3$$

$$a_{21} = -a_{12}$$

$$a_{22} = 1.0000000 - 0.00024976\, T_B^2 - 0.00000015\, T_B^3$$

$$a_{23} = -0.00010859\, T_B^2 - 0.000000030\, T_B^3$$

$$a_{31} = -a_{13}$$

$$a_{32} = a_{23}$$

$$a_{33} = 1.0000000 - 0.00004721\, T_B^2 + 0.000000020\, T_B^3$$

and

$$\Delta a_{11} = -0.00029697\, \tau_2 - 0.000000390\, \tau_3$$

$$\Delta a_{12} = 0.02234988\, \tau_1 + 0.00000676\, \tau_2 - 0.00000663\, \tau_3$$

$$\Delta a_{13} = 0.00971711\, \tau_1 - 0.00000207\, \tau_2 - 0.00000288\, \tau_3$$

$$\Delta a_{21} = -\Delta a_{12}$$

$$\Delta a_{22} = -0.00024976\, \tau_2 - 0.000000450\, \tau_3$$

$$\Delta a_{23} = -0.00010859\, \tau_2 - 0.000000090\, \tau_3$$

$$\Delta a_{31} = -\Delta a_{13}$$

$$\Delta a_{32} = +\Delta a_{23}$$

$$\Delta a_{33} = -0.00004721\, \tau_2 + 0.000000060\, \tau_3$$

and $\tau_1 = \Delta T$

$$\tau_2 = \Delta T\,(2T_B + \Delta T)$$

$$\tau_3 = \Delta T\left(T_B^2 + T_B\,\Delta T + \tfrac{1}{3}\Delta T^2\right)$$

The quantities ΔT and T_B are calculated in the subroutine DATE, described in Section 2.1 below. T_B is the number of days from Jan. 1, 0^h1950 to the base date, divided by 36525. ΔT is the number of days from the base date to the present time, divided by 36525. The base date depends on the launch date of a particular trajectory being Jan 0,0^h of the subsequent year.

The above form of the transformation is derived in Appendix B from the forms given in the literature (see References 9,11,12).

The elements of the $[A]$ matrix are computed whenever needed, except that if $[A]$ has been computed within the previous 1322 seconds, the previous value is used.

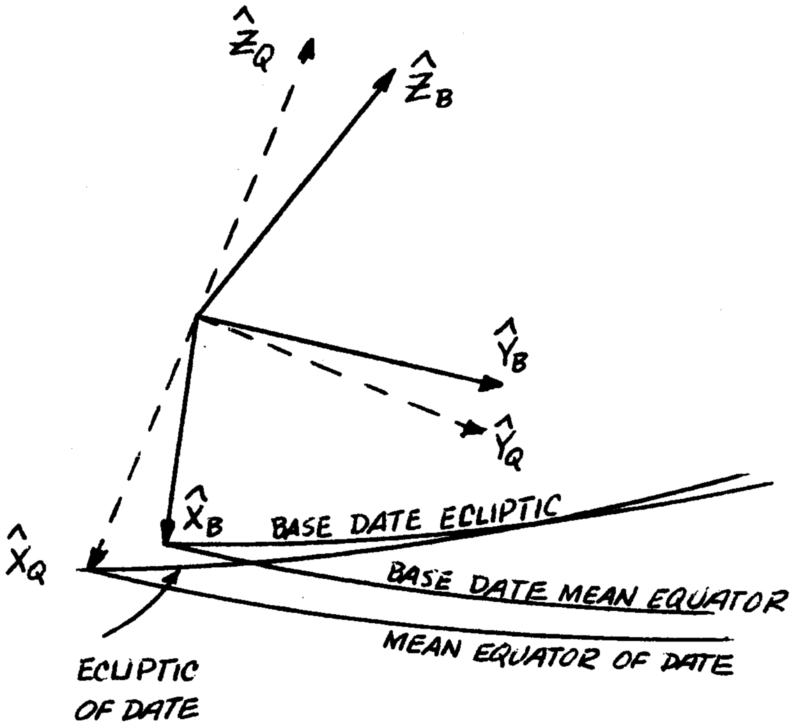


FIGURE A-1
PRECESSION OF EQUINOXES

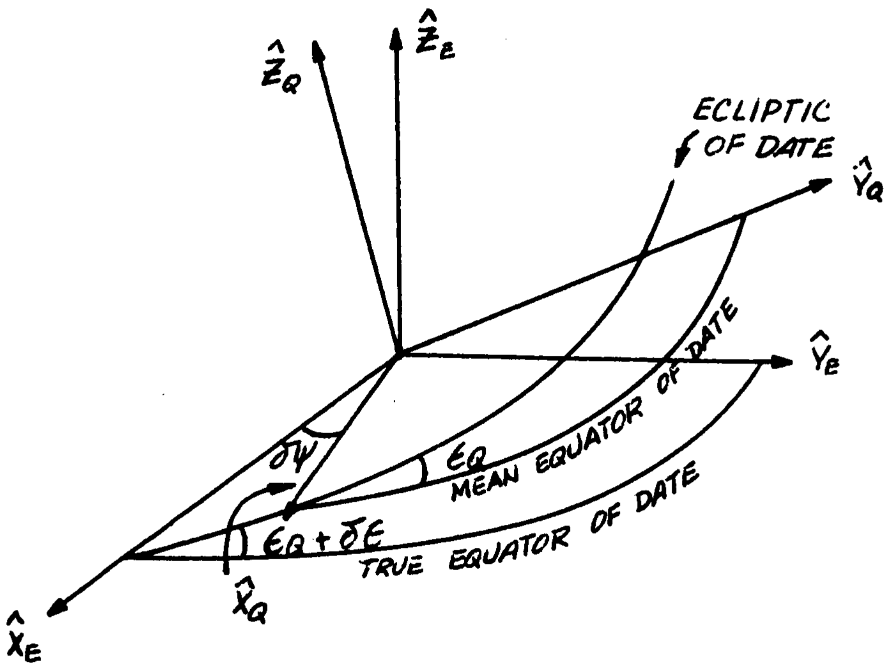
1.2 NUTA: Nutation

The oscillatory motion of the $\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$ system about its mean position ($\hat{X}_Q\ \hat{Y}_Q\ \hat{Z}_Q$) is given by the transformation from $\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$ to $\hat{X}_Q\ \hat{Y}_Q\ \hat{Z}_Q$.

$$[N] = \begin{bmatrix} 1 & -\delta\psi \cos\epsilon_Q & -\delta\psi \sin\epsilon_Q \\ \delta\psi\cos\epsilon_Q & 1 & -\delta\epsilon \\ \delta\psi \sin\epsilon_Q & \delta\epsilon & 1 \end{bmatrix}$$

where $\delta\psi$, $\delta\epsilon$ and ϵ_Q are obtained from subroutine EXPR. The geometric significance of $\delta\psi$, $\delta\epsilon$ and ϵ_Q is shown in Figure A-2.

FIGURE A-2
NUTATION
ANGLES



The [N] matrix above is an approximation, valid to about 0.5×10^{-8}. The exact transformation is given in reference 9, pp 67-68. Complete discussions of nutation may be found in references 14 and 15.

The nutation terms are recomputed if needed and if the prior values are more than 0.1 day old.

1.3 LIBRA: Libration

The transformation from the moon-fixed axis $\hat{X}_M \; \hat{Y}_M \; \hat{Z}_M$ to the $\hat{X}_E \; \hat{Y}_E \; \hat{Z}_E$ axes, pictured in Figure A-3, is given by [L]:

$$[L] = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

where the l's are given in terms of the three angles Ω', Λ, i:

$$l_{11} = \cos\Omega' \cos\Lambda - \sin\Omega' \sin\Lambda \cos i$$

$$l_{12} = -\cos\Omega' \sin\Lambda - \sin\Omega' \cos\Lambda \cos i$$

$$l_{13} = \sin\Omega' \sin i$$

$$l_{21} = \sin\Omega' \cos\Lambda + \cos\Omega' \sin\Lambda \cos i$$

$$l_{22} = -\sin\Omega' \sin\Lambda + \cos\Omega' \cos\Lambda \cos i$$

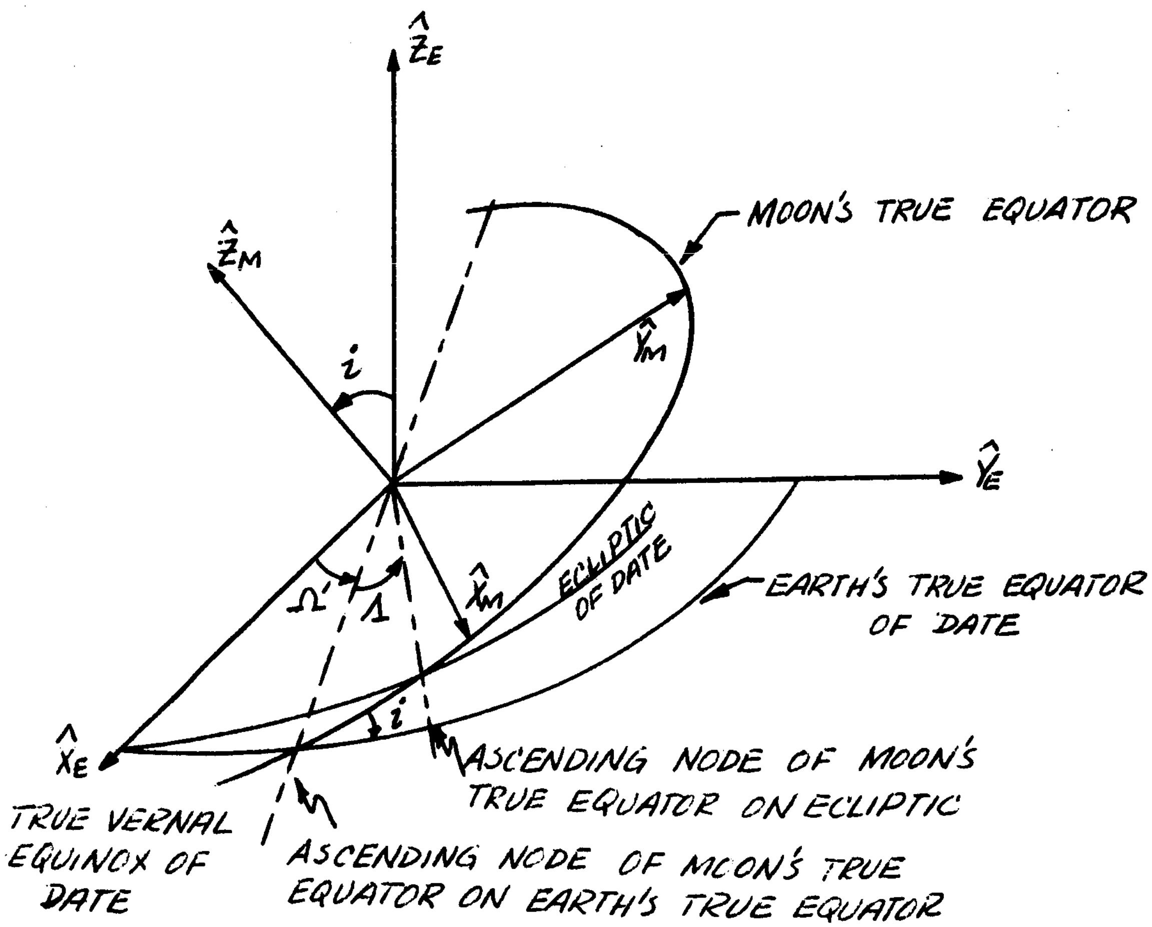
$$l_{23} = -\cos\Omega' \sin i$$

$$l_{31} = \sin\Lambda \sin i$$

$$l_{32} = \cos\Lambda \sin i$$

$$l_{33} = \cos i$$

FIGURE A-3 LIBRATION ANGLES



Ω' = Right Ascension of Ascending Node of Moon's True Equator

Λ = Anomaly from Ascending Node of Moon's True Equator on the Earth's True Equator to the Moon's Axis $\hat{X}_M$.

i = Inclination of Moon's True Equator to Earth's True Equator

The angles Ω', Λ, i are obtained from

$$\sin \Omega' = -\sin(\Omega + \sigma + \delta\psi)\sin(I+\rho)\csc i \qquad -90^\circ < \Omega' + 90^\circ$$

$$\Lambda = \Delta + \mathcal{C} - \Omega + \tau - \sigma \qquad 0^\circ \leq \Lambda \leq 360^\circ$$

$$\cos i = \cos(I+\rho)\cos\epsilon_E + \sin\epsilon_E \sin(I+\rho)\cos(\Omega + \sigma + \delta\psi)$$

$$0^\circ < i < 90^\circ$$

where

$$I = 1^\circ 32.1'$$

$$\sin\Delta = -\sin(\Omega + \sigma + \delta\psi)\csc i \sin\epsilon_E \qquad 0^\circ \leq \Delta < 360^\circ$$

$$\cos\Delta = -\cos(\Omega + \sigma + \delta\psi)\cos\Omega' - \sin(\Omega + \sigma + \delta\psi)\sin\Omega'\cos\epsilon_E$$

$$\sigma = \frac{1}{\sin I}\left[-0.^\circ0302777 \sin g + 0.^\circ0102777 \sin(g + 2\omega) - 0.^\circ00305555 \sin(2g + 2\omega)\right]$$

$$\tau = -0.^\circ003333 \sin g + 0.^\circ0163888 \sin g' + 0.005 \sin 2\omega$$

$$\rho = -0.^\circ0297222 \cos g + 0.^\circ0102777 \cos(g + 2\omega) - 0.^\circ00305555 \cos(2g + 2\omega)$$

and

$$g = 215.^\circ54013 + 13.^\circ064992(d - d_{50})$$

$$g' = 358.^\circ009067 + 0.^\circ9856005(d - d_{50})$$

$$\omega = 196.^\circ745632 + 0.^\circ1643586(d - d_{50})$$

$$\epsilon_E = \epsilon_Q + \delta\epsilon$$

The quantities $\epsilon_Q, \delta\epsilon, \delta\psi, \mathbb{C}, \Omega$ are obtained from the subroutine EXPR., the quantity $(d-d_{50})$ is obtained from the subroutine DATE.

The libration formulas are taken from reference 13, and may be found also in references 9 and 11.

The libration matrix is recomputed when needed, except that the prior values are used if they were calculated less than .01 hours previously.

1.4 GAMMAT: Greenwich Hour Angle of the True Vernal Equinox

The subroutine GAMMAT computes the rotation matrix $[\gamma]$ that transforms a vector from the $\hat{X}_G\ \hat{Y}_G\ \hat{Z}_G$ system to the $\hat{X}_E\ \hat{Y}_E\ \hat{Z}_E$ system. (see section VI-2 for definitions of the two coordinate systems)

$$[\gamma] = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\gamma = \gamma_M + \delta\alpha$ degrees

$$\gamma_M = \begin{bmatrix} 100.^\circ 07554260 + 0.^\circ 9856473460\, d_i \\ + 2.^\circ 9015 \times 10^{-13} (d_i)^2 + 0.^\circ 4178074622 06\, t' \end{bmatrix}_{\text{MODULO } 360^\circ}$$

$$d_i = IP\,[d-d_{50}] \quad (\text{no dimensions})$$

$$t' = [3600(t-t_\ell+HRS) + 60\,HMIN + SEC] - \left\{ IP\left[\frac{3600(t-t_\ell+HRS)+60\,HMIN+SEC}{86400}\right]\right\} 86400$$

$$W_e = \frac{0.004178074\overline{6}2}{(1 + 5.21 \times 10^{-13} d_i)} \text{ degrees/SEC}$$

and $$\delta\alpha = \delta\psi \cos \epsilon_Q$$

$$IP(x) = \text{integral part of } (x)$$

The inputs $\delta\psi$, ϵ_Q are obtained from EXPR, and the inputs $(d-d_{50})$, $(t-t_L)$ are obtained from DATE. The values for HRS, HMIN and SEC are launch date inputs to MINIVAR.

The entire subroutine is programmed in double precision in order to avoid loss of accuracy in γ_M (because of the mod 360°) and in t' (because of subtraction of large, nearly equal numbers).

The expression for γ is given in references 9 and 11.

1.6 <u>GENMAT: General Purpose Orthogonal Transformation</u>

The transformations [G], [DRA] and [S] defined in Section VI-3 all have the same form:

$$[G] = \begin{bmatrix} -\sin\lambda_G & -\sin\phi_G \cos\lambda_G & \cos\phi_G \cos\lambda_G \\ \cos\lambda_G & -\sin\phi_G \sin\lambda_G & \cos\phi_G \sin\lambda_G \\ 0 & \cos\phi_G & \sin\phi_G \end{bmatrix}$$

where λ_G and ϕ_G are geodetic longitude and latitude of the sub-vehicle point or of the observation station.

For [DRA] , the right ascension λ_E of the vehicle or station replaces λ_G , and the declination ϕ_E replaces ϕ_G .

For [S] , the lunar longitude λ_M of the vehicle replaces λ_G and lunar latitude ϕ_M replaces ϕ_G .

The above transformations may be obtained by inspection of Figures A-1, A-2, and A-3.

2.0 Subsidiary Subroutines

2.1 DATE: Subroutine for time quantities

The subroutine DATE produces the time parameters for use in other subroutines. The inputs are càlendar date of launch and time since launch. The output quantities are T , T_B , ΔT and $d-d_{50}$. The program is purely procedural. It is limited to launch dates between 1960 and 1970.

NYEARP	=	year of launch
DAYS	=	day of year at which launch occurs. (example DAYS = 31 for launch on 8:5 Jan 31)
HRS	=	number of hours fully expired from beginning of day of launch to time of launch.
HMIN	=	number of minutes fully expired from beginning of hour of launch to time of launch
SEC	=	number of seconds and fractional parts of a second expired from beginning of minute of launch to time of launch
$t-t_L$	=	hours and fractions thereof from launch time to present time of trajectory calculation.

Procedure:

1. Calculate I: (I) = NYEARP - 1959

2. Lookup YR(I) and YR (I+1) from Table:

(I)	YR(I)
1	3651
2	4017
3	4382
4	4743
5	5112
6	5478
7	5843
8	6208
9	6573
10	6939
11	7304

3. Calculate Output Quantities T_B, $d-d_{50}$, T, ΔT:

$$TB = YR(I+1)/36525 \quad \text{(No dimensions)}$$

$$d-d_{50} = YR(I) + DAYS - 1 + HRS/24 + HMIN/1440$$
$$+ SEC/86400 + (t-t_\ell)/24 \quad \text{(no dimensions)}$$

$$T = (d-d_{50})/36525 \quad \text{(no dimensions)}$$

$$\Delta T = T - T_B \quad \text{no dimensions}$$

These outputs are recalculated as demanded by other subroutines.

2.2 EXPR: Long expressions for $\delta\psi, \delta\epsilon, \epsilon_Q, \Omega, ☾$:

This subroutine takes in T and ($d-d_{50}$) from the subroutine DATE and puts out nutation in longitude $\delta\psi$, nutation in obliquity $\delta\epsilon$, mean obliquity ϵ_Q, mean longitude of descending node of moon's equator on ecliptic Ω and mean longitude of moon ☾, for use in [N],

$[L]$ and $[\dot{L}]$. The entire subroutine is repeated upon demand by other subroutines.

$$\epsilon_Q = 23^\circ.4457874 - 0^\circ.01301376T - 0^\circ.8855\times10^{-6}T^2 + 0^\circ.503\times10^{-6}T^3$$

$$\delta\epsilon = \Delta\epsilon + d\epsilon \quad \text{degrees}$$

$$\begin{aligned}\Delta\epsilon = {} & +0^\circ.255833\times10^{-2}\cos\Omega - 0^\circ.25\times10^{-4}\cos 2\Omega \\ & +0^\circ.1530555\times10^{-3}\cos 2L + 0^\circ.61111\times10^{-5}\cos(3L-\Gamma') \\ & -0^\circ.25\times10^{-5}\cos(L+\Gamma') - 0^\circ.194444\times10^{-5}\cos(2L-\Omega) \\ & -0^\circ.8333\times10^{-6}\cos(2\Gamma'-\Omega)\end{aligned}$$

$$\begin{aligned}d\epsilon = {} & +0^\circ.24444\times10^{-4}\cos 2\mathbb{C} + 0^\circ.5\times10^{-5}\cos(2\mathbb{C}-\Omega) \\ & +0^\circ.30555\times10^{-5}\cos(3\mathbb{C}-\Gamma') - 0^\circ.13888\times10^{-5}\cos(\mathbb{C}+\Gamma') \\ & -0^\circ.8333\times10^{-6}\cos(\mathbb{C}-\Gamma'+\Omega) + 0^\circ.8333\times10^{-6}\cos(\mathbb{C}-\Gamma'-\Omega) \\ & +0^\circ.5555\times10^{-6}\cos(3\mathbb{C}-2L+\Gamma') + 0^\circ.5555\times10^{-6}\cos(3\mathbb{C}-\Gamma'-\Omega)\end{aligned}$$

$$\begin{aligned}\Omega = {} & 12^\circ.1127902 - 0^\circ.0529539222(d-d_{50}) + 0^\circ.20795\times10^{-2}T \\ & +0^\circ.2081\times10^{-2}T^2 + 0^\circ.2\times10^{-5}T^3\end{aligned}$$

$$\begin{aligned}\mathbb{C} = {} & 64^\circ.37545167 + 13^\circ.1763965268(d-d_{50}) - 0^\circ.1131575\times10^{-2}T \\ & -0^\circ.113015\times10^{-2}T^2 + 0^\circ.19\times10^{-5}T^3\end{aligned}$$

$$\delta\psi = \Delta\psi + d\psi \quad \text{degrees}$$

$$\Delta\psi = -\left[0°.47895611\times10^{-2} + 0°.47222\times10^{-5}T\right]\sin\Omega$$
$$+ 0°.580550\times10^{-4}\sin 2\Omega - 0°.35333\times10^{-3}\sin 2L$$
$$+ 0°.350\times10^{-4}\sin(L-\Gamma) - 0°.13888\times10^{-4}\sin(3L-\Gamma)$$
$$+ 0°.58333\times10^{-5}\sin(L+\Gamma) + 0°.3333\times10^{-5}\sin(2L-\Omega)$$
$$+ 0°.13888\times10^{-5}\sin(2\Gamma'-\Omega) + 0°.11111\times10^{-5}\sin(2L-2\Gamma')$$

$$d\psi = -0°.56666\times10^{-4}\sin(2\mathcal{C}) + 0°.18888\times10^{-4}\sin(\mathcal{C}-\Gamma')$$
$$+ 0°.83333\times10^{-6}\sin 2(\mathcal{C}-\Gamma') - 0°.94444\times10^{-5}\sin(2\mathcal{C}-\Omega)$$
$$- 0°.7222\times10^{-5}\sin(3\mathcal{C}-\Gamma') + 0°.41666\times10^{-5}\sin(\mathcal{C}-2L+\Gamma')$$
$$+ 0°.30555\times10^{-5}\sin(\mathcal{C}+\Gamma') + 0°.16666\times10^{-5}\sin 2(\mathcal{C}-L)$$
$$+ 0°.16666\times10^{-5}\sin(\mathcal{C}-\Gamma'+\Omega) + 0°.16666\times10^{-5}\sin(\mathcal{C}-\Gamma'-\Omega)$$
$$- 0°.13888\times10^{-5}\sin(3\mathcal{C}-2L+\Gamma') - 0°.1111\times10^{-5}\sin(3\mathcal{C}-\Gamma'-\Omega)$$

where Γ, Γ', L are obtained from

$$\Gamma = 282°.08053028 + 0°.470684\times10^{-4}(d-d_{50})$$
$$+ 0°.45525\times10^{-3}T + 0°.4575\times10^{-3}T^2$$
$$+ 0°.3\times10^{-5}T^3$$

$$\Gamma' = 208°.8439877 + 0°.1114040803(d-d_{50}) - 0°.010334\,T$$
$$- 0°.0103434\,T^2 - 0°.12\times10^{-4}T^3$$

$$L = 280°.08121009 + 0°.9856473354(d-d_{50})$$
$$+ 0°.302\times10^{-3}T + 0°.302\times10^{-3}T^2$$

These expressions are taken from references 9 and 13; complete discussion of them is given in reference 11 under the explanation of the ephemeris for the sun, and in reference 12.

3.0 Subroutines for Transforming Initial Conditions and for Transforming Oblateness Attractions

3.1 XFORM: Transformation of Initial Conditions

This subroutine calls upon all the previous subroutines as needed to convert vehicle initial position and velocity into the base date system of trajectory calculation. Initial conditions consist of three position coordinates and three velocity components. The velocity need not be specified in the same type of coordinates as position.

Table A-I gives in the left column the various forms that initial position or velocity information may take, and in the right column the calculation required to transfer it to the base date system. It has been assumed in the table that $[\dot{A}]$ and $[\dot{N}]$ are negligible in a single precision program. It may be noted that $[\dot{\gamma}] = [W_e][\gamma]$ where

$$[W_e] = \text{antisymmetric matrix corresponding to } \overline{W_e} \times$$

$$= W_e \begin{bmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{in the } \hat{X}_E, \hat{Y}_E, \hat{Z}_E \text{ basis}$$

and W_e is from subroutine GAMMAT.

Also,

$$C = a_e (1 - E^2 \sin^2 \phi_G)^{-1/2}$$

TABLE A-I
TRANSFORMATIONS FOR INITIAL CONDITIONS

VARIOUS FORMS FOR INITIAL CONDITIONS	INITIAL CONDITIONS $X_B\ Y_B\ Z_B\ \dot{X}_B\ \dot{Y}_B\ \dot{Z}_B$ IN BASE DATE SYSTEM
(1) λ_E Right Ascension(RA) ϕ_E Declination (D) R_E Geocentric Dist.	$[A][N][DRA]\begin{bmatrix} 0 \\ 0 \\ R_E \end{bmatrix}$
(2) $V_{\lambda E} = \overrightarrow{V_{VC}}'$ along $\hat{\lambda}_E$ $V_{\phi E} = \overrightarrow{V_{VC}}'$ along $\hat{\phi}_E$ $V_{RE} = \overrightarrow{V_{VC}}'$ along $\hat{R}_E$	$[A][N]\left\{[DRA]\begin{bmatrix} V_{\lambda E} \\ V_{\phi E} \\ V_{RE} \end{bmatrix} + [W_e][DRA]\begin{bmatrix} 0 \\ 0 \\ R_E \end{bmatrix}\right\}$
(3) V magnitude of $\overrightarrow{V_{VC}}'$ γ_E flt.pth. angle $\overrightarrow{V_{VC}}'$ A_E azimuth angle $\overrightarrow{V_{VC}}'$	$[A][N]\left\{[DRA]\begin{bmatrix} V\cos\gamma_E \sin A_E \\ V\cos\gamma_E \cos A_E \\ V\sin\gamma_E \end{bmatrix} + [W_e][DRA]\begin{bmatrix} 0 \\ 0 \\ R_E \end{bmatrix}\right\}$
(4) λ_G Geodetic Longitude ϕ_G Geodetic Latitude h_G Geodetic Altitude	$[A][N][\gamma]\left\{[G]\begin{bmatrix} 0 \\ 0 \\ h_G + C \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \varepsilon^2 C \sin\phi_G \end{bmatrix}\right\} \triangleq [A][N][\gamma]\overrightarrow{R_G}$
(5) $V_e = \overrightarrow{V_{VC}}'$ along east $V_n = \overrightarrow{V_{VC}}'$ along north $V_h = \overrightarrow{V_{VC}}'$ along vertical(up)	$[A][N]\left\{[\gamma][G]\begin{bmatrix} V_e \\ V_n \\ V_h \end{bmatrix} + [W_e][\gamma]\overrightarrow{R_G}\right\}$

TABLE A-I continued

VARIOUS FORMS FOR INITIAL CONDITIONS	INITIAL CONDITIONS $X_B\ Y_B\ Z_B\ \dot{X}_B\ \dot{Y}_B\ \dot{Z}_B$ IN BASE DATE SYSTEM
(6) V magnitude of $\vec{V}_{VC}$ γ_G flight path angle A_G azimuth	$[A][N]\left\{[\gamma][G]\begin{bmatrix} V\cos\gamma_G \sin A_G \\ V\cos\gamma_G \cos A_G \\ V\sin\gamma_G \end{bmatrix} + [W_e][\gamma]\vec{R}_G\right\}$
(7) $X_E = \vec{R}_{VC}'$ along $\hat{X}_E$ $Y_E = \vec{R}_{VC}'$ along $\hat{Y}_E$ $Z_E = \vec{R}_{VC}'$ along $\hat{Z}_E$	$[A][N]\begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}$
(8) $\dot{X}_E = \vec{V}_{VC}'$ along $\hat{X}_E$ $\dot{Y}_E = \vec{V}_{VC}'$ along $\hat{Y}_E$ $\dot{Z}_E = \vec{V}_{VC}'$ along $\hat{Z}_E$	$[A][N]\left\{\begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \end{bmatrix} + [W_e]\begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}\right\}$

$\vec{R}_{VC}'$ = vector from center of reference body to vehicle, expressed in any basis.

$\vec{V}_{VC}'$ = velocity of vehicle relative to (rotating) reference body, expressed in any basis.

Note: It is also possible to input $X_B\ Y_B\ Z_B\ \dot{X}_B\ \dot{Y}_B\ \dot{Z}_B$ directly.

Note: " $\vec{R}_{VC}'$ along $\hat{X}_E$ ", etc., means the dot product $\vec{R}_{VC}' \cdot \hat{X}_E$, etc. In the dot product, $\hat{X}_E$ is expressed in the same basis as chosen for $\vec{R}_{VC}'$.

where a_e (earth radius) and ε^2 (eccentricity) are input to the program as ERAD and EPSSQ.

The initial position may be input in kilometers (KM) or earth radii (ER) for distances and degrees (DG) for angles. The initial velocity may be input in kilometers per second (K/S) or earth radii per hour (ER/HR). Since the trajectory calculation is done in earth radii and earth radii per hour, conversion from KM and K/S to ER and ER/HR is performed. It is performed in subroutine INPUT, which precedes subroutine XFORM.

3.2 OBLATE: Calculation of Oblateness Attractions

3.2.1 Earth Oblateness Attraction

The subroutine OBLATE transforms the vehicle position vector $\vec{R}_{VC}$ relative to the earth's center into the true earth system $\hat{X}_E \; \hat{Y}_E \; \hat{Z}_E$. The oblateness attraction given in equations 6 and 7 of Section IV is then calculated using the transformed values of $\vec{R}_{VC}$. The resultant attraction vector is then expressed in the base date system. The formulas employed are, in effect,

$$(\vec{F}_Z)_B = [A][N]\{ b(z_E)\, \vec{R}_E + c(z_E)\, \hat{Z}_E \}$$

where

$$\vec{R}_E = [N]^{-1}[A]^{-1}\vec{R}_B \quad , \quad z_E = \hat{Z}_E \cdot \vec{R}_E \quad \text{and} \quad b(z_E),$$

$c(z_E)$ are given in equation 7 with z_E for z and R_B for r.

The vector $(\vec{F}_Z)_B$ is the earth oblateness attraction, due to the nutating, precessing earth, expressed in the base date system. It is used in place of $\vec{F}_Z$ of equation 6 and is equal to $\vec{F}_Z$ if precession and nutation are ignored.

3.2.2 Lunar Oblateness Attraction

Circumlunar trajectories are strongly influenced in the neighborhood of the moon by the lunar oblateness mass. Acceleration terms corresponding to lunar oblateness attraction were added as $\vec{F_4}$ to equation 1:

$$\vec{F_4} \begin{cases} = 0 \quad \text{if } |\vec{R_M}| \geq 40{,}000 \text{ KM.} \quad \text{Otherwise} \\ = -\frac{5}{2}\gamma' \frac{\vec{R_M}}{R_M^7}\left\{\vec{R_M} \cdot [V_M]_B \cdot \vec{R_M}\right\} + \gamma' \frac{\vec{R_M}}{R_M^5} \cdot [V_M]_B \quad (KM/SEC^2) \end{cases}$$

where

$$\gamma' = 0.6671 \times 10^{-19} \; KM^3/(KG-SEC^2)$$

$$\vec{R_M} = \vec{R_{VC}} - \vec{R_{ME}} \quad (KM)$$

$$[V_M]_B = [A][N][L] \begin{bmatrix} 7.3 & 0 & 0 \\ 0 & 1.9 & 0 \\ 0 & 0 & -9.2 \end{bmatrix} [L]^{-1}[N]^{-1}[A]^{-1} \times 10^{25} \; (KG-KM^2)$$

The vector $\vec{R_{VC}}$ is vehicle position relative to earth in the base date system, obtained from the trajectory calculation. The vector $\vec{R_{ME}}$ is moon position relative to the earth, obtained from core storage as XME, YME, ZME in earth radii. In phase II of the orbit determination program $\vec{R_{VC}}$ may be vehicle position relative to a referency body (c) other than earth. In that case the equation for $\vec{R_M}$ is $\vec{R_M} = \vec{R_{VC}} + \vec{R_{CE}} - \vec{R_{ME}}$. In either case the lunar oblateness term is appreciable only if R_M is less than about 40,000 KM.

The previous value for $[V_M]_B$ is used if it was computed not more than 840 sec. prior to demand.

The dimensions of $\vec{F_4}$ as shown above are KM/SEC^2. They are converted to ER/HR^2 before addition to the other perturbation terms.

The derivation of the equations for $\vec{F_4}$ are given in Appendix B.

4.0 List of Inputs and Outputs for Subroutines

Subroutine	Inputs	Outputs
PREC:	$T_B, \Delta T$	$[A]$
NUTA:	$\delta\psi, \delta\epsilon, \epsilon_Q$	$[N]$
LIBRA:	$\delta\psi, \delta\epsilon, \epsilon_Q, \mathcal{C}, \Omega, (d-d_{50})$	$[L], i, \Delta, \sigma, g, g', \omega$
GAMMAT:	HRS, HMIN, SEC $\delta\psi, \epsilon_Q, (d-d_{50}), (t-t_0)$	$[\gamma]$
GENMAT λ_G, ϕ_G	λ_G, ϕ_G	$[G]$
GENMAT λ_E, ϕ_E	λ_E, ϕ_E	$[D, RA]$
GENMAT λ_M, ϕ_M	λ_M, ϕ_m	$[S]$
DATE	NYEARP, DAYS, HRS, HMIN, SEC	$T_B, T, \Delta T, (d-d_{50})$
EXPR	$T, (d-d_{50})$	$\delta\psi, \delta\epsilon, \epsilon_Q, \Omega, \mathcal{C}$
XFORM	a_e, ϵ^2 $\lambda_E, \phi_E, R_E, V_{\lambda E}, V_{\phi E}, V_{RE}$ $V, \gamma_E, A_E, \lambda_G, \phi_G, h_G$ $V_e, V_n, V_h, V, \gamma_G, A_G$ $X_E, Y_E, Z_E, \dot{X}_E, \dot{Y}_E, \dot{Z}_E$ $X_M, Y_M, Z_M, \dot{X}_M \dot{Y}_M \dot{Z}_M$ $\lambda_M, \phi_M, R_M, V_{\lambda M}, V_{\phi M}, V_{RM}$ V_M, γ_M, A_M $[A], [N], [DRA], [\gamma], [G], [L]$	$\vec{R_B}, \vec{V_B}$
OBLATE:	$[A], [N], \vec{R_{VC}}$ $[A], [N], [L], \vec{R_{VC}}, \vec{R_{CE}}, \vec{R_{ME}}$	$\vec{F_2}$ $\vec{F_4}$

APPENDIX B

DERIVATIONS AND ALTERNATE SUBROUTINES

1.0 Derivation of Precession Transformations to a Variable Base Date

The standard form of the precession matrix is a set of elements a_{ij} (T) that are functions of the time T in Julian centuries of 36525 days from a standard time, $0^h.0$ January 1, 1950, to the present epoch. This transformation $[a(T)]$ takes a vector from the present-time system to the January 1, 1950 system, i.e., through the small angle that the earth has precessed in the time T.

It is desired to refer vectors to some recent base date (say $0^h.0$ January 0 of year subsequent to launch) rather than the 1950 date. Let

T_B = time in Julian centuries from $0^h.0$ January 1, 1950 to $0^h.0$ January 0, of year after launch.

ΔT = time in Julian centuries from $0^h.0$ January 0, of year after launch, to the present (trajectory) time.

$T = T_B + \Delta T$ = time in Julian centuries from $0^h.0$ January 1,1950 to trajectory time.

With the above notation, the desired transformation from the present system to the new base date is the product of $[a(T)]$, which transforms from the present date system to the 1950 date system, and $[a(T)]^{-1}$ which goes from the 1950 system to the base date system. Thus

$$[A(\Delta T)] \triangleq [a(T_B)]^{-1} [a(T)]$$

$$= [a(T_B)]^{-1} [a(T_B + \Delta T)]$$

Since $a_{ij}'(T_B+\Delta T)$ is a polynomial of third degree in $(T_B+\Delta T)$ it is a simple matter to write it as a sum:

$$[a(T_B+\Delta T)] = [a(T_B)] + [\Delta a(\Delta T, T_B)]$$

So that

$$[A(\Delta T)] = [a(T_B)]^{-1}\{[a(T_B)] + [\Delta a(T_B,\Delta T)]\}$$
$$= [I] + [a(T_B)]^{-1}[\Delta a(T_B,\Delta T)]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}_{T_B} \begin{bmatrix} \Delta a_{11} & \Delta a_{12} & \Delta a_{13} \\ \Delta a_{21} & \Delta a_{22} & \Delta a_{23} \\ \Delta a_{31} & \Delta a_{32} & \Delta a_{33} \end{bmatrix}_{T_B,\Delta T}$$

where the elements a_{ij}' are $a_{ij}'(T_B)$, the standard forms of the precession transformation elements evaluated at T_B.

The $\Delta a_{ij}''$ are obtained from their definition as follows:

$$a_{ij}(T_B+\Delta T) \triangleq a_{ij}(T_B) + \Delta a_{ij}(\Delta T, T_B)$$

$$\Delta a_{ij}(\Delta T, T_B) = a_{ij}'(T_B+\Delta T) - a_{ij}(T_B)$$
$$= \overset{0}{a}_{ij} + \overset{1}{a}_{ij}(T_B+\Delta T) + \overset{2}{a}_{ij}(T_B+\Delta T)^2 + \overset{3}{a}_{ij}(T_B+\Delta T)^3$$
$$- \overset{0}{a}_{ij} - \overset{1}{a}_{ij}T_B - \overset{2}{a}_{ij}T_B^2 - \overset{3}{a}_{ij}T_B^3$$
$$= \overset{1}{a}_{ij}\Delta T + \overset{2}{a}_{ij}(2T_B\Delta T + \Delta T^2)$$
$$+ \overset{3}{a}_{ij}(3T_B^2\Delta T + 3T_B\Delta T^2 + \Delta T^3)$$
$$= \overset{1}{a}_{ij}\tau_1 + \overset{2}{a}_{ij}\tau_2 + 3\overset{3}{a}_{ij}\tau_3$$

where $\tau_1 \triangleq \Delta T$, $\tau_2 \triangleq (+2T_B\Delta T + \Delta T^2)$, and

$\tau_3 \triangleq (T_B^2 \Delta T + T_B \Delta T^2 + \frac{1}{3}\Delta T^3)$

$$a_{ij}(x) \triangleq \overset{o}{a}_{ij} + a^1_{ij}x + a^2_{ij}x^2 + a^3_{ij}x^3$$

The expanded forms for a_{ij} and Δa_{ij} are given in Appendix A-1.1, PREC.

2.0 <u>Derivation of Triaxial Lunar Oblateness Terms in Base Date System</u>

Let $\vec{R}_M$ be vehicle position relative to moon's center

dM be element of lunar mass

$\vec{r}$ be position of dM relative to moon's center

γ' be universal gravitation constant

M be total mass of moon

Then

V_M = lunar potential

$$= -\int_M \gamma' dM \frac{1}{|\vec{R}_M - \vec{r}|}$$

$$= -\gamma' \int_M dM \frac{1}{R_M}\left[1 - \frac{2\vec{r}\cdot\vec{R}_M}{R_M^2} + \frac{r^2}{R_M^2}\right]^{-1/2}$$

$$= -\gamma' \int_M \frac{dM}{R_M}\left[1 + \frac{\vec{r}\cdot\vec{R}}{R_M^2} - \frac{1}{2}\frac{r^2}{R_M^2} + \frac{3}{4}\left(\frac{2\vec{r}\cdot\vec{R}_M}{R_M^2}\right)^2 + \dots\, \mathcal{O}\left(\frac{r^3}{R_M^3}\right)\dots\right]$$

$$\cong \frac{-\gamma'}{R_M}\left[M + 0 - \frac{1}{2}\left(\int \frac{r^2 dM}{R_M^2} - 3\hat{R}_M \cdot \int \frac{\vec{r}\vec{r}}{R_M^2} dM \cdot \hat{R}_M\right)\right]$$

$$\cong -\frac{\gamma' M}{R_M} + \frac{\gamma'}{R_M^3}\frac{1}{2}\hat{R}_M \cdot \left[\int_M (r^2 \hat{\hat{I}} - 3\vec{r}\vec{r})dM\right] \cdot \hat{R}_M$$

$$\cong \frac{-\gamma' M}{R_M} - \frac{1}{2}\frac{\gamma'}{R_M^5}\vec{R}_M \cdot [V_M] \cdot \vec{R}_M$$

where $[V_M]$, the lunar oblateness dyadic is independent of vehicle position and may be written in terms of lunar constants in a principal lunar axis coordinate system as:

$$[V_M]_M = - \begin{bmatrix} 2A-B-C & 0 & 0 \\ 0 & 2B-A-C & 0 \\ 0 & 0 & 2C-A-B \end{bmatrix}$$

where

$A = \int_M dm\,(Y^2+Z^2)$ = principal moment of inertia on $\hat{X}_M$.

$B = \int_M dm\,(X^2+Z^2)$ = principal moment of inertia on $\hat{Y}_M$.

$C = \int_M dm\,(Y^2+X^2)$ = principal moment of inertia on $\hat{Z}_M$.

The (attractive) force on the vehicle due to the oblateness portion of the lunar potential is $\vec{F_4}$:

$$\vec{F_4} = -\nabla\left\{-\tfrac{1}{2}\gamma' R_M^{-5}\,\vec{R_M}\cdot[V_M]\cdot\vec{R_M}\right\}$$

$$= \tfrac{1}{2}\gamma'\left\{\nabla(R_M^{-5})\,\vec{R_M}\cdot[V_M]\cdot\vec{R_M} + R_M^{-5}\,\nabla\left[\vec{R_M}\cdot[V_M]\cdot\vec{R_M}\right]\right\}$$

Now

$$\nabla(R_M^{-5}) = -5R_M^{-6}\,\nabla R_M = -5R_M^{-6}\,\frac{\vec{R_M}}{R_M} = \frac{-5}{R_M^{7}}\,\vec{R_M}$$

$$\nabla\left\{\vec{R_M}\cdot[V_M]\cdot\vec{R_M}\right\} = \nabla\left\{\vec{R_M}\cdot\begin{bmatrix} V_{11} & 0 & 0 \\ 0 & V_{22} & 0 \\ 0 & 0 & V_{33}\end{bmatrix}\cdot\vec{R_M}\right\}$$

$$= \nabla\left\{V_{11}X_M^2 + V_{22}Y_M^2 + V_{33}Z_M^2\right\}$$

$$= 2V_{11}X_M\hat{X}_M + 2V_{22}Y_M\hat{Y}_M + 2V_{33}Z_M\hat{Z}_M$$

$$= 2[V_M]\cdot\vec{R_M}$$

where $\vec{R_M}$ was written in the cartesian system in which $[V_B]$ is diagonal. Thus $\vec{F_4}$ becomes

$$\vec{F_4} = -\frac{5}{2}\frac{\gamma'}{R_M^7}\vec{R_M}\ \vec{R_M}\cdot[V_M]\cdot\vec{R_M} + \frac{\gamma'}{R_M^5}[V_M]\cdot\vec{R_M}$$

where $\vec{R_M}$ is assumed to be written in the same base system as $[V_M]$. Since $\vec{R_M}$ is the result of the trajectory calculation, it will be available in the base date $\hat{X}_B, \hat{Y}_B, \hat{Z}_B$ system. It is therefore necessary to write $[V_M]$ in that system. This may be done by the similarity transformation composed of precession, nutation and libration:

$$[V_M]_B = [A][N][L][V_M]_M\ [L]^{-1}[N]^{-1}[A]^{-1}$$

The values of γ' A,B,C employed are those given in reference 9, p.79:

$$\begin{aligned}
\gamma' &= 0.6671\times10^{-19}\ KM^3/KG\text{-}SEC^2\\
A &= 0.88746\times10^{29}\quad KG\text{-}KM^2\\
B &= 0.88764\times10^{29}\quad KG\text{-}KM^2\\
C &= 0.88801\times10^{29}\quad KG\text{-}KM^2
\end{aligned}$$

3.0 Alternate Subroutine DATE

The following equations will accept a calendar launch date and time and produce the required $T_B, T, \Delta T$ and $(d-d_{50})$ for launch dates from A.D.1950 to 2000. The inputs and outputs are identical to those described under DATE, Appendix A-2.2 except that instead of DAYS, the following two quantities are input:

$D\ell$ = day of month
$M\ell$ = month of year.

For example, launch on January 30, 1963 at 2:30 P.M. would be input as NYEARP 1963.0 , $M_\ell = 1.0$, $D_\ell = 30.0$, HRS = 14.0, HMIN = 30.0

(1) $d_B - d_\ell$ = integral number of days from 0.0 of launch date to base date, counting launch day but not base day.

$$= 365 + \lambda - (D_\ell - 1) - IP(\beta) - \delta$$

where

$$\alpha = (NYEARP - 1948)/4.0 + 0.01$$

$$\beta = 30.608\, M_\ell - 32.408$$

$$\lambda = \begin{cases} 1 & \text{if } [\alpha - IP(\alpha)] < 0.01 \text{ and } M_\ell < 3 \\ 0 & \text{Otherwise} \end{cases}$$

$$\delta = \begin{cases} IP(1.6\, M_\ell) & \text{if } M_\ell < 2.5 \\ 0 & \text{if } M_\ell > 2.5 \end{cases}$$

(2) $t_B - t_\ell$ = time from launch to base date, in hours

$$= 24.0(d_B - d_\ell) - HRS - \frac{HMIN}{60} - \frac{SEC}{3600}$$

(3) $$\Delta T = \frac{(t - t_\ell) - (t_B - t_\ell)}{(24.0)(36525)}$$

= time from base date to time of trajectory in Julian centuries of 36525 days.

(4) $$T_B = \frac{365.0(NYEARP - 1949) + IP(\alpha)}{36525}$$

= Julian centuries from $0^h.0$ Jan 1, 1950 to base date

(5) $$T = T_B + \Delta T$$

(6) $$(d - d_{50}) = 36525.0(T)$$

APPENDIX C

EQUATIONS FOR PROPAGATION CORRECTION

1.0 Computation of Ray Bending

Referring to Figure C-1, consider a ray entering at angle β an infinitesimal layer of thickness $d\rho$. Since the curvature of the ray is equal to the component of the refractive gradient normal to the ray, divided by the index of refraction, it follows that:

$$\frac{1}{K} = \frac{1}{n} \frac{dn}{d\rho} \cos\beta \tag{1}$$

where K is the radius of curvature.

The length of the ray path in the layer is

$$K d\gamma = \csc\beta \, d\rho \tag{2}$$

which, combined with (1), gives

$$d\gamma = \frac{1}{n} \frac{dn}{d\rho} \cot\beta \, d\rho \tag{3}$$

Since $d\gamma$'s of all elementary layers are directly additive, as shown in Figure C-1, by considering $d\gamma'$ due to bending between points Q and R, it follows that the contribution to the total bending γ, due to a layer bounded by the heights ρ_j' and ρ_k is

$$\gamma_{jk} = \int_{\rho_j'}^{\rho_{k}'} \frac{1}{n} \frac{dn}{d\rho} \cot\beta \, d\rho \tag{4}$$

If the ray departs from the earth's surface with the elevation angle of θ_o Snell's Law for spherical stratification states:

$$n_0 a \cos\theta_0 = n\rho\cos\beta = \text{constant} \tag{5}$$

where

n_0 = surface index of refraction,

a = Earth's radius

$\rho = a + h$

h = height above earth,

n = index of refraction at the specified height,

From (5)

$$\cos\beta = (n_0 a / n\rho)\cos\theta_0 = (n_j \rho_j / n\rho)\cos\beta_j \tag{6}$$

$$\begin{aligned} \sin\beta &= (n_0 a / n\rho)[(n\rho/n_0 a)^2 - \cos^2\theta_0]^{1/2} \\ &= (n_j \rho_j / n\rho)[(n\rho/n_j\rho_j)^2 - \cos^2\beta_j]^{1/2} \end{aligned} \tag{7}$$

$$\begin{aligned} \cot\beta &= [(n\rho/n_0 a)^2 - \cos^2\theta_0]^{-1/2}\cos\theta_0 \\ &= [(n\rho/n_j\rho_j)^2 - \cos^2\beta_j]^{-1/2}\cos\beta_j \end{aligned} \tag{8}$$

where n, ρ and β are the values of these parameters at some height h.

Equation (8) can be substituted in (4) to give the general equation for refractive bending

$$\begin{aligned} \gamma_{jk} &= \int_{\rho_j}^{\rho_k} \frac{1}{n} \frac{dn}{d\rho} \frac{\cos\theta_0}{[(n\rho/n_0 a)^2 - \cos^2\beta_j]^{1/2}} \, d\rho \\ &= \int_{\rho_j}^{\rho_k} \frac{1}{n} \frac{dn}{d\rho} \frac{\cos\beta_j}{[(n\rho/n_j\rho_j)^2 - \cos\beta_j]^{1/2}} \, d\rho \end{aligned} \tag{9}$$

It is assumed that a) $dn/d\rho = -k$ = constant, b) $\rho_k - \rho_j \ll \rho_j$, and c) index of refraction n is very nearly equal to unity. On the basis of these assumptions

$$k = \frac{(N_j - N_k) \times 10^{-6}}{\rho_k - \rho_j} = \frac{(N_j - N) \times 10^{-6}}{\rho - \rho_j} \tag{10}$$

where

$$N = (n-1)10^6$$

$$\begin{aligned} (n\rho/n_j\rho_j)^2 &= \{[1-(N_j-N)10^{-6}][1+(\rho-\rho_j)/\rho_j]\}^2 \\ (n\rho/n_j\rho_j)^2 &\cong 1 + 2(\rho-\rho_j)(1-k\rho_j)/\rho_j \end{aligned} \tag{11}$$

and, substituting in (9)

$$\begin{aligned} \gamma_{jk} &= k \cos\beta_j \int_{\rho_j}^{\rho_k} [\sin^2\beta_j + 2(\rho-\rho_j)(1-k\rho_j)/\rho_j]^{-1/2} d\rho \\ &= \frac{k\rho_j \cos\beta_j}{1-k\rho_j} \{[\sin^2\beta_j + 2(\rho_k-\rho_j)(1-k\rho_j)/\rho_j]^{1/2} - \sin\beta_j\} \end{aligned} \tag{12}$$

From (8), (9), and (11)

$$\begin{aligned} \sin\beta_k &= (n_j\rho_j/n_k\rho_k)[(n_k\rho_k/n_j\rho_j)^2 - \cos^2\beta_j]^{1/2} \\ &= \frac{\cos\beta_k}{\cos\beta_j}[\sin^2\beta_j + 2(\rho_k-\rho_j)(1-k\rho_j)/\rho_j]^{1/2} \end{aligned} \tag{13}$$

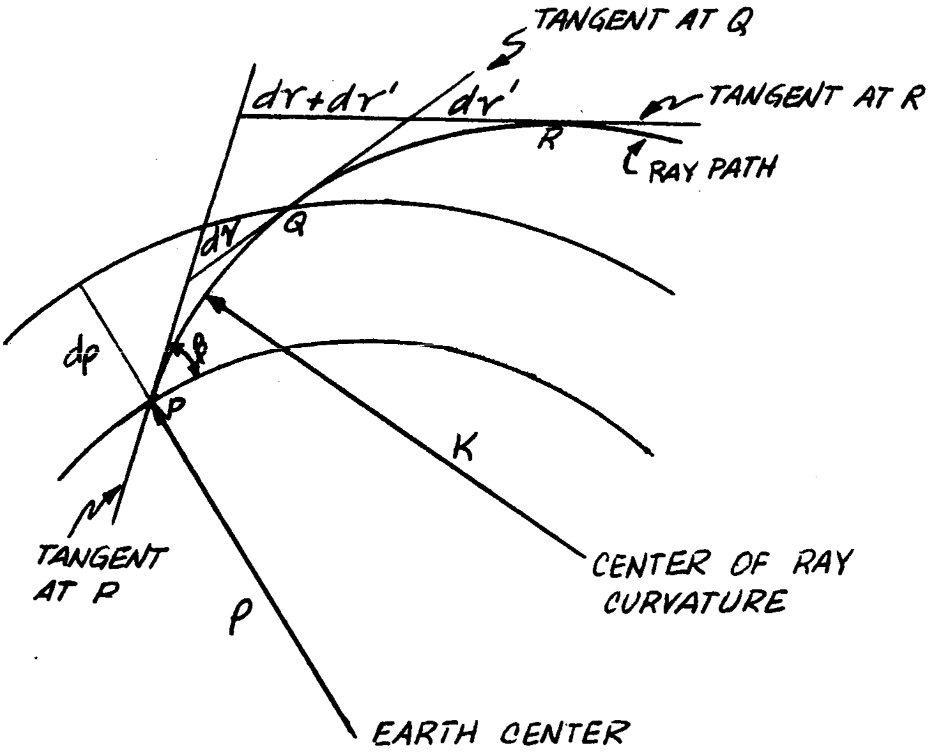


Figure C-1 Geometry of Bending Through an Infinitesimal Layer

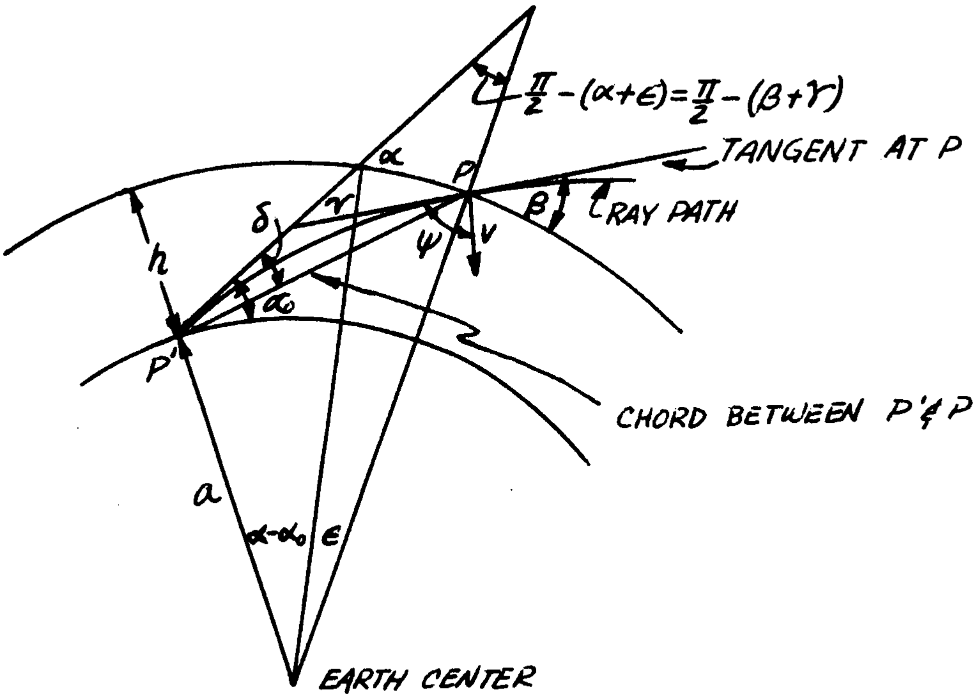


Figure C-2 Geometry of Bending Through a Refractive Layer

Combining with (12)

$$\gamma_{jk} = \frac{kP_j \cos^2 \beta_j}{1 - kP_j} \left(\tan \beta_k - \tan \beta_j \right) \tag{14}$$

From (8), (10), and (11)

$$\frac{kP_j}{1-kP_j} = \frac{2(N_j - N_k)\, 10^{-6} \sec^2 \beta_j}{\sec^2 \beta_k - \sec^2 \beta_j}$$

$$= \frac{2(N_j - N_k)\, 10^{-6} \sec^2 \beta_j}{\tan^2 \beta_k - \tan^2 \beta_j} \tag{15}$$

which, substituted in (14), gives the desired expression for γ_{jk}:

$$\gamma_{jk} = \frac{(N_j - N_k)\, 10^{-6}}{\frac{1}{2}(\tan \beta_j + \tan \beta_k)}$$

$$= \frac{N_j - N_k}{500(\tan \beta_j + \tan \beta_k)} \text{ milliradians} \tag{16}$$

where N is expressed in N units, $(n - 1) \times 10^6$.

Total bending through the atmosphere is simply the sum of individual contributions.

$$\gamma(mr) = \sum_{i=1}^{m_k} \frac{(N_{i-1} - N_i)}{500(\tan \beta_{i-1} + \tan \beta_i)} \tag{17}$$

It is frequently convenient to measure the refractive error in terms of the angle subtended from the earth's center. This quantity, denoted by ϵ, can be readily obtained from Figure C-2.

$$\epsilon = \gamma - (\theta - \beta) \tag{18}$$

The quantity $(\theta - \beta)$ may be found in the following manner. From Snell's law,

$$n_0 \cos\theta = n \cos\beta$$

or

$$\cos\beta = \cos[\theta - (\theta - \beta)] \tag{19a}$$
$$= [1 + (N_0 - N)10^{-6}]\cos\theta$$

$$\cos\theta = \cos[\beta + (\theta - \beta)] \tag{19b}$$
$$= [1 - (N_0 - N)10^{-6}]\cos\beta$$

Expansion of (19) and the application of small angle approximations results in

$$(\theta - \beta) = \{1 - [1 - 2(N_0 - N)10^{-6}\cot^2\theta]^{1/2}\}\tan\theta \tag{20a}$$

$$= \{[1 + 2(N_0 - N)10^{-6}\cot^2\beta]^{1/2} - 1\}\tan\beta \tag{20b}$$

At heights above the troposphere for rays departing tangentially, or for angles of elevation greater than 100 milliradians at any height, θ and β are very nearly equal and (20) reduces to:

$$(\theta - \beta) \cong (N_0 - N)10^{-6}\cot\theta \tag{21}$$
$$\cong (N_0 - N)10^{-6}\cot\beta$$

2.0 Computation of Errors in Principal Measurements

2.1 Elevation Angle Error

In most practical applications the quantity of greatest interest is the elevation angle error. This quantity, denoted by δ, can be obtained from Figure C-2 by the use of the law of sines.

$$a \cos \theta_0 = \rho \cos \theta$$
$$a \cos(\theta_0 - \delta) = \rho \cos[(\theta + \epsilon) - \delta] \tag{22}$$

From (22)

$$\tan \delta = \frac{\sin \epsilon \tan \theta + (1 - \cos \epsilon)}{\sin \epsilon + \cos \epsilon \tan \theta - \tan \theta_0}$$

or

$$\delta = \frac{\epsilon \tan \theta + \epsilon^2/2}{\epsilon + \tan \theta - \tan \theta_0} \tag{23}$$

Omitting $\epsilon^2/2$ in the numerator of (23) results in an error of about five per cent in the troposphere for a tangentially departing ray. At higher angles of elevation or greater heights, this error becomes negligible.

It should be noted that whereas τ and ϵ, due to the passage of the ray through various layers, are directly additive, δ's are not. Thus, to evaluate δ at ionospheric heights or above, it is first necessary to combine the tropospheric and the ionospheric ϵ's or δ's. However, it turns out that in nearly all practical cases above the troposphere $\frac{\epsilon^2}{2} \ll \epsilon$ and $\epsilon \ll (\tan \theta - \tan \theta_0)$; consequently the omission of $\epsilon^2/2$ in the numerator and ϵ in the denominator usually results in less than five percent error at F region heights.

Equation (23) may thus be approximated by

$$\delta = \frac{\epsilon \tan\theta}{\tan\theta - \tan\theta_0}$$

It is, therefore, usually justifiable to add directly the tropospheric and ionospheric δ's to obtain the total elevation angle error.

At astronomical distances all three quantities (γ, ϵ and δ) become numerically equal.

2.2 Retardation of the Signal Passing Through a Region of a Constant Refractive Gradient

Signal retardation $d\tau$ caused by a layer of thickness $d\rho$ (Figure C-1) is given by

$$\begin{aligned} d\tau &= \left(\frac{1}{v} - \frac{1}{c}\right) \csc\beta \, d\rho \\ &= (c/v - 1) \csc\beta \, d\rho / c = N \times 10^{-6} c^{-1} \csc\beta \, d\rho \end{aligned} \tag{24}$$

where c and v are signal velocities in free space and the medium, respectively.

The range error is given by

$$\Delta r_{jk} = \int_{\rho_j}^{\rho_k} c \, d\tau = \int_{\rho_j}^{\rho_k} \frac{N \times 10^{-6} d\rho}{\sin\beta} \tag{25}$$

In evaluating γ, from equation 16,

$$\gamma_{jk} = \int_{\rho_j}^{\rho_k} \frac{(dn/d\rho)\,d\rho}{\tan\beta} = \int_{\rho_j}^{\rho_k} \frac{dn}{\tan\beta} = \frac{(N_j - N_k) \times 10^{-6}}{\frac{1}{2}(\tan\beta_j + \tan\beta_k)} \qquad (26)$$

In other words, the value of the integral for the case of a constant radial gradient was found to be very nearly equal to the one that would have been obtained had the average value of the denominator of the integrand been taken as a constant. The integral of (23) can be treated in a similar manner. Furthermore, at low angles sine and tangent are nearly the same and at high angles the rate of change of sine is so slow that such procedure is certainly justifiable.

Thus (25) is evaluated by setting

$$\Delta r_{jk} = \int_{\rho_j}^{\rho_k} \frac{N \times 10^{-6}}{\sin\beta}\,d\rho = \frac{2 \times 10^{-6}}{\sin\beta_j + \sin\beta_k} \int_{\rho_j}^{\rho_k} N\,d\rho$$

From (10)

$$\int_{\rho_j}^{\rho_k} N\,d\rho = \int_{\rho_j}^{\rho_k} [N_j - k(\rho - \rho_j)]\,d\rho$$

$$= N_j(\rho_k - \rho_j) - \frac{1}{2}(N_j - N_k)(\rho_k - \rho_j)$$

$$= \frac{1}{2}(N_j + N_k)(\rho_k - \rho_j)$$

Substituting in (25)

$$\Delta r_{jk} = \frac{(N_k - N_j)(\rho_k - \rho_j) \times 10^{-6}}{\sin\beta_k + \sin\beta_j} \tag{27}$$

To compute retardation for a double passage through the layer, (27) must be doubled. The resulting final formula for Δr is:

$$\Delta r = \frac{2}{10^3} \sum_{i=1}^{m} \frac{|N_{i+1} + N_i|(h_i - h_{i-1})}{\sin\beta_{i-1} + \sin\beta_i} \quad \text{meters} \tag{28}$$

In the ionosphere the formula for range propagation error is

$$\Delta r = \frac{1 + \left(\frac{f_2}{f_1}\right)^2}{10^6} \sum_{i=1}^{m} \frac{|N_{i-1} + N_i|(h_i - h_{i-1})}{\sin\theta_{i-1} + \sin\theta_i} \tag{29}$$

where f_1 = up frequency

f_2 = down frequency

2.3 Doppler Error

Due to the refractive bending, there will generally be an error in the measurement of the radial component of the target velocity. The equation describing this can readily be derived with the aid of Figure C-2.

Let

$\underline{R}$ = station location vector in inertial coordinates

$\underline{r}$ = position vector from earth's center to satellite in inertial coordinates

$\underline{\rho}'$ = position vector from station to satellite in inertial coordinates

$\underline{\rho}$ = position vector from station to satellite in topocentric local moving coordinates

$\underline{\Omega}$ = earth's rotation velocity vector in inertial coordinates

A = Coordinate Conversion transformation matrix

$\underline{i}', \underline{j}', \underline{k}'$ = Unit vectors, inertial coordinate system

$\underline{i}, \underline{j}, \underline{k}$ = Unit vectors, topocentric moving coordinate system

Therefore

$$\underline{r} = X\underline{i}' + Y\underline{j}' + Z\underline{k}'$$
$$\dot{\underline{r}} = \dot{X}\underline{i}' + \dot{Y}\underline{j}' + \dot{Z}\underline{k}'$$
$$\dot{\underline{R}} = \underline{\Omega} \times \underline{R}$$
$$\underline{\rho}' = \underline{r} - \underline{R}$$
$$\dot{\underline{\rho}}' = \dot{\underline{r}} - \dot{\underline{R}} = \dot{\underline{r}} - \underline{\Omega} \times \underline{R}$$
$$\underline{\rho} = A\underline{\rho}' = A(\underline{r} - \underline{R})$$
$$\dot{\underline{\rho}} = A\dot{\underline{\rho}}' = A(\dot{\underline{r}} - \underline{\Omega} \times \underline{R})$$

Let
$$\frac{\underline{\rho}'}{|\underline{\rho}'|} = \underline{i} = A_{11}\underline{i}' + A_{12}\underline{j}' + A_{13}\underline{k}'$$

$$\frac{\underline{\rho}' \times \underline{R}}{|\underline{\rho}' \times \underline{R}|} = \underline{k} = A_{31}\underline{i}' + A_{32}\underline{j}' + A_{33}\underline{k}'$$

$$\underline{k} \times \underline{i} = \underline{j} = A_{21}\underline{i}' + A_{22}\underline{j}' + A_{23}\underline{k}'$$

$$A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}$$

$$\therefore \dot{\underline{\rho}} = A\dot{\underline{\rho}}' = \dot{\rho}_x\underline{i} + \dot{\rho}_y\underline{j} + \dot{\rho}_z\underline{k}$$

Here $\dot{\rho}_x$ = velocity component along the local range vector

$\dot{\rho}_y$ = velocity component normal to the local range vector in a plane determined by the transmitter beam and the earth's center

$\dot{\rho}_z$ = velocity component normal to a plane determined by the transmitter beam and the earth's center.

From the diagram the measured value of range rate is along the apparent path or along the tangent to the path at the satellite.

$$V \text{ measured} = \dot{\rho}_x \cos(\gamma - \delta) - \dot{\rho}_y \sin(\gamma - \delta)$$

$$V \text{ radial} \doteq \dot{\rho}_x$$

Therefore the range rate error is

$$\Delta V_r \doteq V \text{ radial} = V \text{ measured}$$
$$= \dot{\rho}_x - \dot{\rho}_x \cos(\gamma - \delta) + \dot{\rho}_y \sin(\gamma - \delta)$$

and since $(\gamma - \delta)$ is a very small angle

$$\Delta V_r = (\gamma - \delta)\dot{\rho}_y$$

The above quantity is doubled for a roundtrip error.

In the ionosphere, the above correction for range-rate is modified as follows

$$\Delta V_r \approx \left[1 + \left(\frac{f_2}{f_1}\right)^2\right](\gamma - \delta)\dot{\rho}_y$$

3.0 Computation of Errors in Secondary Angular Measurements

The elevation angle error, computed in the preceding section, must be transformed into the coordinate system of the secondary angular measurements in order to determine the equivalent error in these systems.

3.1 Coordinate Conversions

To convert from the azimuth angle (ϕ), elevation angle (θ) system to other systems:

From Figure C-3B the following relations hold for the X,Y angles:

$$\sin Y = \cos\theta \cos\phi$$

$$\left.\begin{aligned} \cos Y \sin X &\doteq \cos\theta \sin\phi \\ \cos Y \cos X &\doteq \sin\theta \end{aligned}\right\} \longrightarrow \tan x = \operatorname{ctn}\theta \sin\phi$$

also from Figure C-3B the following relations hold for the l,m direction cosines.

$$l \doteq \cos\theta \sin\phi$$

$$m = \cos\theta \cos\phi$$

From Figure C-3B the following relations hold for the hour angle, declination angles

$$\sin d \doteq \sin\theta \sin\lambda + \cos\theta \cos\phi \cos\lambda$$

$$\cos d \sin h \doteq \cos\theta \sin\phi$$

$$\cos d \cos h \doteq \cos\theta \cos\phi \sin\lambda - \sin\theta \cos\lambda$$

$$\therefore \tan h \doteq \frac{\sin\phi}{\cos\phi \sin\lambda - \tan\theta \cos\lambda}$$

3.2 The Error Components

A small deviation in the elevation angle will cause a small deviation in the secondary angles. Then magnitudes can be determined by simply differentiating the previously determined coordinate conversion expressions with respect to the elevation angle.

For the X - Y system

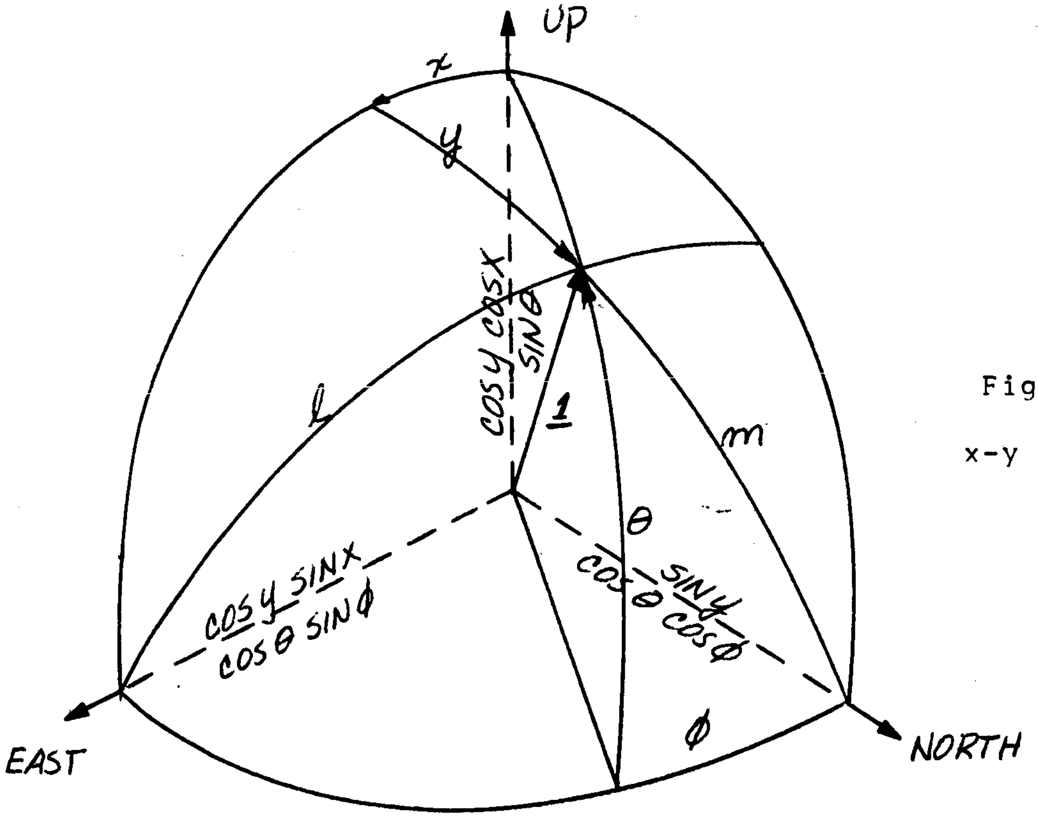


Figure C-3A

x-y and l-m System

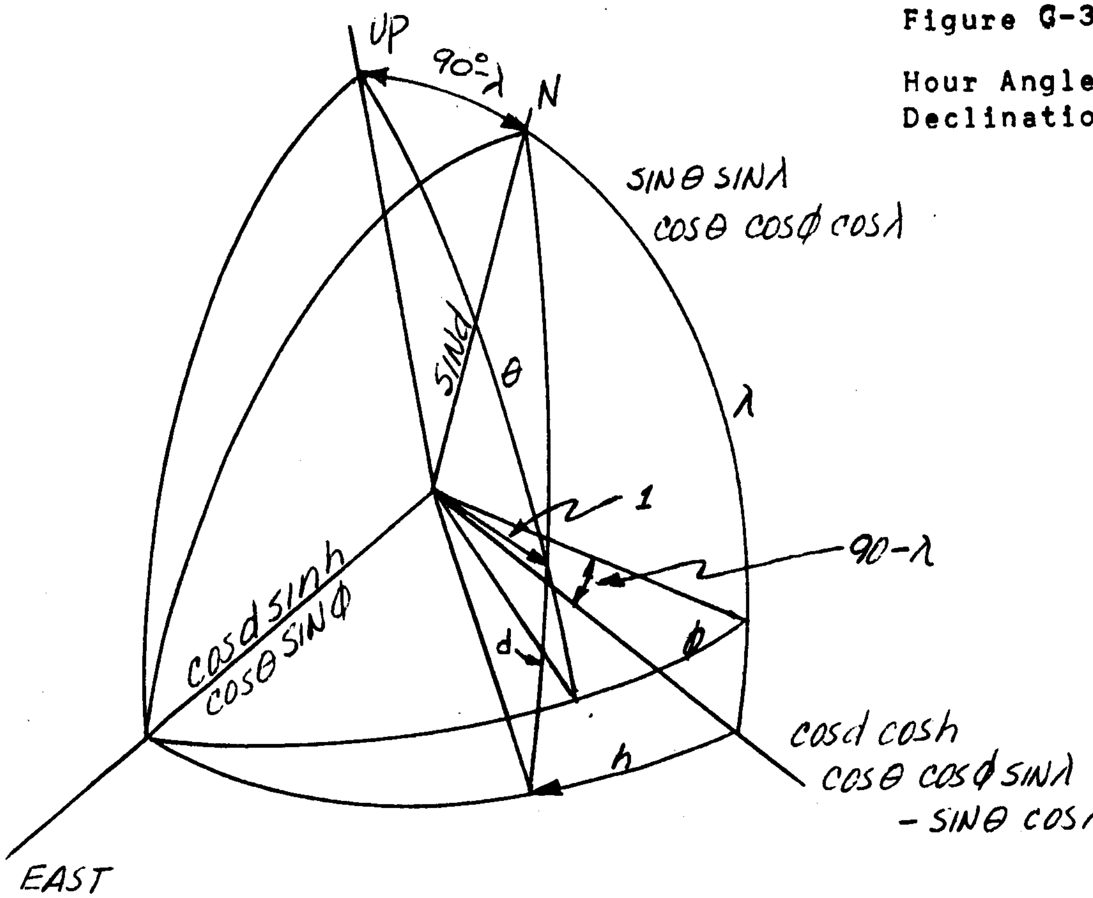


Figure G-3B

Hour Angle - Declination System

$$\frac{\partial x}{\partial \theta} = \frac{1}{SEC^2 x}\left(-SIN\phi\, CSC^2\theta\right) = \frac{-SIN\phi\, COS^2 x}{SIN^2\theta}$$

$$\frac{\partial Y}{\partial \theta} = \frac{1}{COS\, Y}\left(-COS\phi\ SIN\theta\right)$$

$$\therefore\ \Delta X = \frac{\partial x}{\partial \theta}\delta \quad \text{and} \quad \Delta Y = \frac{\partial y}{\partial \theta}\delta$$

where δ is the elevation angle error found in the preceding section.

For the l - m system

$$\frac{\partial l}{\partial \theta} = -SIN\theta\ SIN\phi$$

$$\frac{\partial m}{\partial \theta} = -SIN\theta\ COS\phi$$

$$\Delta l = \frac{\partial l}{\partial \theta}\delta \qquad \Delta m = \frac{\partial m}{\partial \theta}\delta$$

For the hour angle - declination system

$$\frac{\partial d}{\partial \theta} = \frac{COS\theta\ SIN\lambda - SIN\theta\ COS\phi\ COS\lambda}{COS\, d}$$

$$\frac{\partial h}{\partial \theta} = \frac{1}{SEC^2 h}\left[\frac{SIN\phi\ COS\lambda\ SEC^2\theta}{(COS\phi\ SIN\lambda - TAN\theta\ COS\lambda)^2}\right]$$

$$= \frac{SIN\phi\ COS\lambda\ COS^2 h}{(COS\theta\ COS\phi\ SIN\lambda - SIN\theta\ COS\lambda)^2}$$

APPENDIX D

AMBIGUITY RESOLUTION IN THE GODDARD RANGE AND RANGE RATE SYSTEM

The Goddard Range and Range Rate system measures range with a CW phase measuring system whose output is ambiguous (i.e., the exact value is uncertain without the use of outside information). The system allows exact ranging within ambiguous one-way ranges of 11,000, 2750 or 550 miles (depending on the selected range tone) but does not contain sufficient information to determine the multiple of the ambiguous range in which the vehicle lies.

The range ambiguity is resolved by integrating to a value of time close to the time of the data point, then using a two body solution to get an estimate (R_I) of the range. The use of the two-body solution provides a simplified method for determining the nominal range by bypassing problems with numerical integration which arise if the exact trajectory computation were employed.

The unambiguous range, Ra, is dependent on the lowest range, f_L, used and is given by

$$Ra = c/2f_L \tag{1}$$

The data from the $R/\dot{R}$ system include a control digit which specifies the lowest range tone used to measure the range, either 8, 32, or 160 cycles per second. The correct (unambiguous) round trip propagation time, T_{RT}, excluding the effects of transponder delay and propagation is:

$$T_{RT} = T_P + \left[\frac{R_I}{Ra}\right] T_a \tag{2}$$

where T_P is the measured time interval between corresponding zero crossings of the transmitted and received signal (R in the data block) and Ta is the period of the lowest range tone:

$$T_a = 1/f_L \tag{3}$$

The brackets, [ ], in Equation 2 mean the integer portion of the quotient R_I/Ra. The true range, Rc, is then given by

$$R_c = \frac{c}{2} T_{RT} \tag{4}$$

In order to insure that small errors in R_I do not cause the wrong number of periods of the lowest range tone to be added to the indicated range a reasonableness check is made:

$$R_I - \frac{1}{2}\left(\frac{c}{2} T_a\right) \leq T_{RT} \cdot \frac{c}{2} \leq R_I + \frac{1}{2}\left(\frac{c}{2} T_a\right) \tag{5}$$

If this inequality is not satisfied an appropriate change in the integer R_I/Ra is made.

APPENDIX E

INCLUSION OF BIAS ERRORS

A method for including the effects of bias errors in the instrumentation of the covariance matrix describing vehicle position and velocity uncertainty is presently being considered. The method does not assume that the bias is considered as an additional state; rather, it considers the statistical aspects only. When this work is completed, it will be documented in this appendix.

APPENDIX F

TREATMENT OF THE PLANETARY COORDINATES

A tape of solar, lunar, and planetary coordinates based on Naval Observatory data is employed. It is in the form of overlapping two-year files, i.e., 1963-64, 1964-65, etc., with the coordinates of the various bodies referred to the mean equinox and equator of the middle of the two year file. The subroutine EPHEM searches the tape and reads in the proper file and record, keeping 30 days of tables in core storage at a time.

The first record on each file consists of the year in fixed decimal. Each of the subsequent records contain the following information:

Word 1 : Initial time of record in hours from base time.

Then 12 consecutive 15 word blocks containing the equatorial coordinates of:

XSE	YSE	ZSE	Sun wrt Earth
XJS	YJS	ZJS	Jupiter wrt Sun
XAS	YAS	ZAS	Mars wrt Sun
XVS	YVS	ZVS	Venus wrt Sun

Then three-30 word blocks containing:

XME	YME	ZME	Moon wrt Earth

The Moon coordinates are stored in half-day intervals (0.0^h, $12^h.0$ ET), and the unit of distance is the radius of the Earth. All other tables are in daily intervals (0^h.E.T.), the unit of distance being the AU.

At present, an ephemeris tape is available for 1961-1970, written in nine, two-year files. The files overlap one year.

The astronomical tables are stored in core in 24 hour intervals for the Sun and the planets and 12 hour intervals for the Moon. There are always 30 days of tables available arranged in such a way that the value of time for which the interpolation takes place is not near either end of the table. In Earth reference, the sequence of coordinates in the tables, all referred to the Earth as origin, is as follows:

In location "Table" there is the time of the first entry from the initial time. In "Table 1-30" there are 30 x coordinates of the Sun.

In "Table 31" to "Table 60", the y coordinates of the Sun.

In "Table 61" to "Table 90", z coordinates of the Sun.

In "Table 91" to "Table 180", the x, y and z coordinates of Jupiter

In "Table 181" to "Table 270", the x, y, z coordinates of Mars.

In "Table 271" to "Table 360", the x, y, z coordinates of Venus.

In "Table 361" to "Table 420", the x coordinates of the Moon.

In "Table 421" to "Table 480", the y coordinates of the Moon and in "Table 481" to "Table 540", z coordinates of the Moon.

APPENDIX G

CONSTANTS USED IN PROGRAM

CONSTANT	EQUIVALENCE	REPRESENTS	VALUE	UNITS
CONJR	$3/2\ \mu\ J_{20}$	Second harmonic of earth's potential	$.32321941 \times 10^{-1}$	ER^3/HR^2
CONAR	$-\mu\ J_{30}$	third harmonic of earth's potential	$.45791657 \times 10^{-4}$	$(ER)^3/(HR)^2$
CONKR	$-\frac{15}{4}\ \mu\ J_{40}$	fourth harmonic of earth's potential	.1343886	$(ER)^3/(HR)^2$
HMU	μ	Gravitational parameter times earth's mass	19.9094165	$(ER)^3/HR^2$
	J_{20}	not used directly	1082.3×10^{-6}	-
	J_{30}	not used directly	-2.3×10^{-6}	-
	J_{40}	not used directly	-1.8×10^{-6}	-
ERAD	Re	earth's equatorial radius	$6.378165 \times 10^{+3}$	KM
EPSSQ	ϵ^2	ellipticity of earth, squared	$.6693422 \times 10^{-2}$	-
AMERAD	A.U.	astronomical unit	23455	E.R.
-	-	degrees to radian conversion	57.2957795131	°/rad
-	e	Velocity of light	299792.5	KM/S
	μ_{SUN}	GM_{SUN}	$.33295729 \times 10^{6}$	
	μ_{MOON}	GM_{MOON}	$.1229483 \times 10^{-1}$	
	μ_{VENUS}	GM_{VENUS}	.8147689	
	μ_{MARS}	GM_{MARS}	.10782	
	$\mu_{JUPITER}$	$GM_{JUPITER}$	$.317887 \times 10^{+3}$	

Data Handling

APPENDIX H

DATA HANDLING ROUTINES

1.0 Introduction

Three separate methods are available for the editing and combining of data prior to entering the main program. This section discusses these three modes and gives program details involved. The major problems encountered here are: 1. editing of raw $R/\dot{R}$ system data; 2. sorting, merging, and ordering. Any other program can be used if its output has data time ordered and in the proper format.

2.0 Operational Modes

2.1 Preferred Mode

2.1.1 Form of Data

$R/\dot{R}$ - Thirty days of raw $R/\dot{R}$ system data is available. The data are in sections, each section corresponding to a pass of the satellite over a station. The sections are not necessary time ordered.

Minitrack - The ℓ and m direction cosine data are available on one tape, the data being in sections corresponding to a pass of the satellite over a station, but not being necessarily time ordered. The data have already passed through the Goddard CDC160A computer for editing, WWV correction, etc.

2.1.2 Format

See Appendices I and J for format of the data as they are made available.

2.1.3 Data Flow

The following diagram illustrates the data flow in this mode.

2.1.4 Required Programs

2.1.4.1 <u>SORT 1</u>

2.1.4.1.1 Purpose

This routine does a preliminary editing of the raw R & $\dot{R}$ data (available in the format described in 2.1.1 and 2.1.2 above), and places the edited data on a single tape which is used by the subsequent IBSYS SORT program.

2.1.4.1.2 Requirements

a. Each tape has up to 3 R & $\dot{R}$ stations data

b. Each tape has an end of file at the end of the data

c. A set of data consists of two records, each 50 characters long

FORMAT of line 1

$$XXXXXQ_1RRRRRRRRQ_2\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}.DDDHHMMSSQ_3RRRRRRRRQ_4\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}b$$

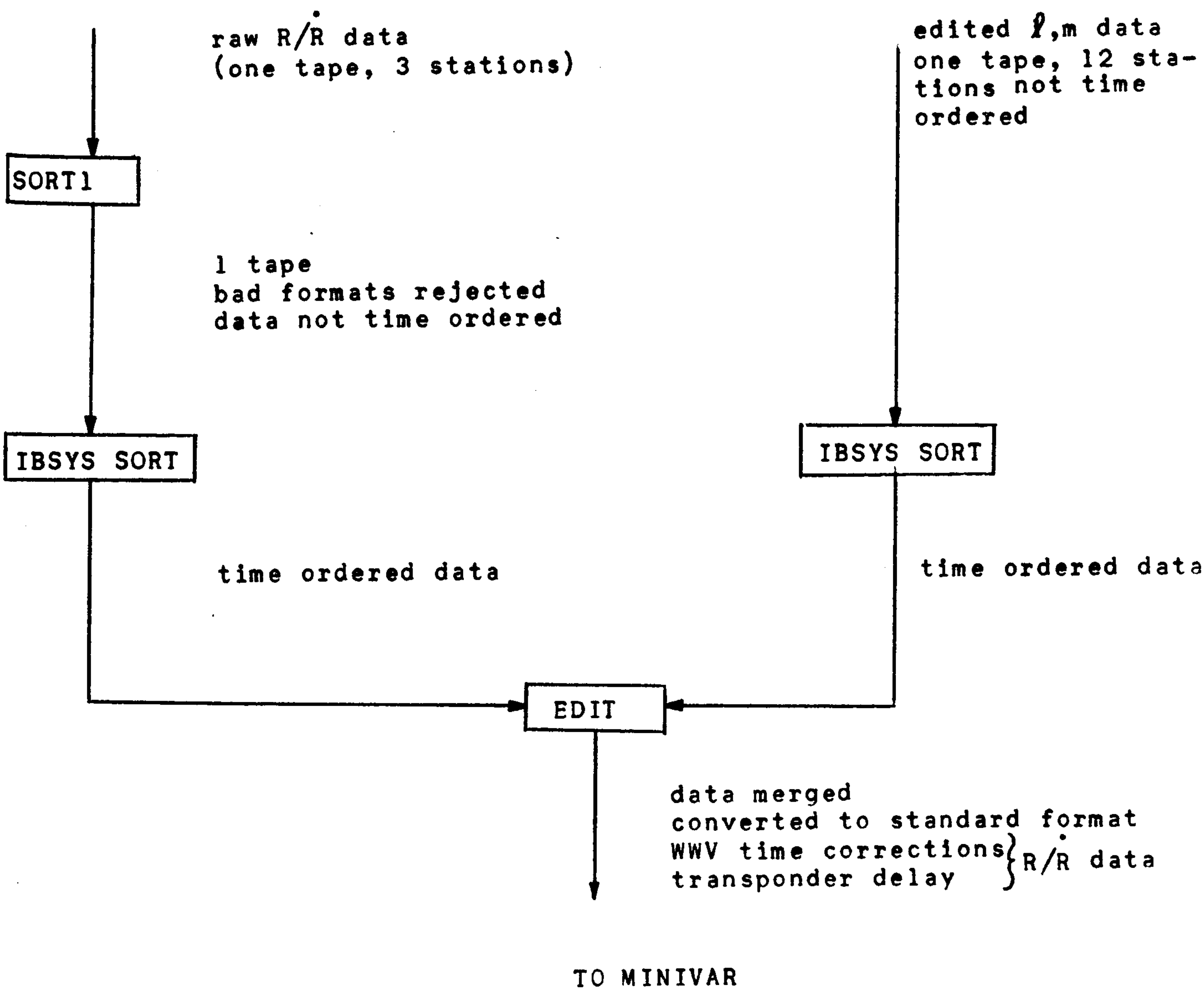
FORMAT of line 2

$$YYYYYQ_5RRRRRRRRQ_6\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}/\begin{matrix}SSSSS\\AAATT\\TTTAA\end{matrix}C_1C_2C_3C_4Q_7RRRRRRRRQ_8\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}b$$

where:

XXXXX signed (1st digit can be blank if value is positive) 4 digit number where decimal point is assumed between 3rd and 4th "X". It defines the X angle.

Q_I One character quality bit. A space indicates good data. Any non-space character (including zero) indicates bad data. This quality bit refers to the data immediately following it.


raw $R/\dot{R}$ data
(one tape, 3 stations)
edited ℓ,m data
one tape, 12 stations not time ordered
SORT1
1 tape
bad formats rejected
data not time ordered
IBSYS SORT
IBSYS SORT
time ordered data
time ordered data
EDIT
data merged
converted to standard format
WWV time corrections
transponder delay
$R/\dot{R}$ data
TO MINIVAR

RRRRRRRR 8 digit Δ t of range

$\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}$ 7 digit Δ t' of range rate doppler counter

DDD integral number of days of year measurement was taken

HH integral number of hours of day measurement was taken (Universal Time)

MM integral number of minutes of hour measurement was taken

SS integral number of seconds of minute measurement was taken

Note: The recorded time is the integral number of seconds corresponding approximately to the first data point. The timing of the following data points are discussed in Appendix I.

b space

YYYYY defines Y angle of range antenna (same format as XXXXX).

SSS
AAA
TTT satellite name or ID

SS
TT
AA Station number (64 is Scottsdale,
26 is Rosman)

C_1 1 digit ambiguity and resolution indicator

C_2 1 digit recording rate and punch ID

C_3 1 digit S-band or VHF system indicator

C_4 1 digit range counter frequency and angular tracking mode indicator

For a more complete definition of these elements, see Appendix I.

2.1.4.1.3 Method

Each data point is read in, the bad formats are rejected, and the output format is caused to conform to the following section on output.

2.1.4.1.4 Data Rejection

a. line identification ("." or "/") not present

b. 50th character of each line not a space

c. last $\dot{R}$ digit of either line is a space

d. any non-numeric information in X,Y,R,$\dot{R}$,D,H,M,S,STA, C_1,C_2,C_3,C_4 data fields

e. STA data do not correspond to any input station number

2.1.4.1.5 Input

CARD	VALUE	COL	PURPOSE
1	ANAME	2-72	Any ID or other information desired for printing
2	NN	5	Number of input raw data tapes (max of 3)
	N	10	Number of different input stations
	ISTA(1)	11-15	MINIVAR station number
	ISTA1(1)	16-20	corresponding $\dot{R}$-R station number
	ISTA(2)	21-25	MINIVAR station number
*	ISTA1(2)	26-30	corresponding $\dot{R}$-R station number
	ISTA(3)	31-35	MINIVAR station number
	ISTA1(3)	36-40	corresponding $\dot{R}$-R station number

*Only use as many sets of ISTA-ISTA1 as specified by N.

Raw data tapes B5, B6, B7 (starting at B5). It is not necessary to mount scratch tapes on unused drives.

2.1.4.1.6 Output

Data written in BCD on B-8 in following format per record (96 characters)

$$\text{DDDHHMMSSXXX.}\dot{\text{XXRRRRRRRRRRRRRR}}\text{RRRRRRRRRRRR}\dot{\text{RRRRRRR}}\text{YYY.YYRRRRRRRRRRRR}\dot{\text{RRRRRRR}}$$

$$\text{RRRR}\dot{\text{RRRRRRR}}\text{RRRRRR}Q_1Q_2Q_3Q_4Q_5Q_6Q_7Q_8C_1C_2C_3C_4\text{SSS*}$$

*S = ISTA(I)

In addition, a count of the number of points written is output on A3 at end of run.

2.1.4.1.7 Subroutines Required

2.1.4.1.7.1 MASK - a FAP subroutine

<u>Purpose</u>: Test a required bit for being a space, a period, a slash or a numerical digit.

<u>Usage</u>:

CALL MASK (BIT, I, IANS, IFLAG)

where

BIT = bit being tested

I = 0 should be space
1 should be a numerical digit
-1 should be "."
-2 should be "/"

IANS - Used only when I = 1; value of BIT as fixed point integer is retained in this location

IFLAG 0 - required condition met
1 - condition not met
-1 - (only used when I = 1) bit not a number

2.1.4.1.7.2 EOFIX - a FAP subroutine

<u>Purpose</u>: avoid termination by EXEM when an end of file or illegal character is encountered in a read statement.

<u>Usage</u>:

Required program sequence:

```
      CALL EOFIX (IND)
      IF (IND) 1,2,3
2     READ---
      CALL EXEMR
      .
      .
      .
1     CALL EXEMR
      .
      .
      .
3     CALL EXEMR
      .
      .
      .
```

where

```
IND  0 - normal condition
     1 - (in address location of word) EOF encountered in READ
    -1 - (in address location of word) illegal character en-
                                       countered in READ
```

The "EXEMR" resets EXEM to its normal condition.

2.1.4.2 EDIT

2.1.4.2.1 Purpose

Merge and put in standard format (i.e. format acceptable to MINIVAR) the data from up to 6 separate types of systems.

2.1.4.2.2 Requirements

1. All data are assumed to be time ordered outside of any 10 points, i.e., the 11th point will not precede the 1st point in any consecutive group of 11 points. (If this condition is not met, see separate write-up for IBM IBSYS SORT routine)

2. Range-Range Rate data are in format produced by SORTST or SORT.

3. MINITRACK data have the format illustrated by Appendix. J.

Note: At present, the only systems available are MINITRACK and RANGE-RATE. Therefore, MERC, DSIFA, DSIFB, and SPARE are dummy subroutines.

2.1.4.2.3 Method

Ten data points from each system tape specified are read into core. The lowest point is chosen and written on the master output tape. A point to replace the written point is then read in from the same system tape that the written point came from. The lowest point is again chosen and the process repeated until all points have been processed.

2.1.4.2.4 Input

CARD	VALUE	COL	PURPOSE
1	ANAME	2-72	Any ID or other information desired
2	N	1-5	number of different systems to be merged
	ITYPE (1)	6-10	system ID
	ITYPE (2)	11-15	
	ITYPE (3)	16-20	
	ITYPE (4)	21-25	
	ITYPE (5)	26-30	

Only specify as many ITYPE's as N calls for.

Type of System	ID Number
MINITRACK	1
MERCURY	2
DSIF A	3
DSIF B	4

Type of System	ID Number
RANGE-RANGE RATE	5
SPARE	6

At present only 1 and 5 can be used.

The following input refers to the various systems and must be sequenced in the same order as ITYPE(I) is assigned.

Minitrack input:

CARD	VALUE	COL	PURPOSE
1'	NUM	1-5	Number of different MINITRACK stations
2'	STA(1)	1-6	MINITRACK station name exactly as it appears on the data
	ISTA1(2)	7-11	Corresponding MINIVAR Station number
3'	STA(2)		
	ISTA1(2)		

Use as many cards of STA and ISTA1 as specified by NUM.

R-RDOT Input:

CARD	VALUE	COL	PURPOSE
1"	NN	1-5	Number of diff. R-R stations
	IYRR	6-10	Year R-R data were taken
	TT	11-28	Vehicle transponder delay in hours
2"	KK	1-5	Minivar station number
	WWV(KK)	6-23	WWV delay for that station in hours
3"	KK		
	WWV(KK)		

Use as many sets of KK, WWV(KK) as specified by NN

Tape assignments:

MINITRACK - A5

MERCURY - B5

DSIF A - B7

DSIF B - A10

RANGE-RANGE RATE - B8

SPARE - B9

Note: A5 or B8 must be mounted with scratch tapes if the respective system isn't used.

2.1.4.2.5 Output

B6 written in binary. Each record will consist of 21 words. Data will have following format:

Word 1,2,3	Time in hours, minutes, and seconds respectively from Jan. 1, 0.0 hr., 1960
Word 4	Station number (MINIVAR input data must correspond with this)
Word 5	Types of data
Word 6-Word 21	Actual Observations

With regard to words 6-21, the data are packed into the low order words and trailing words are filled with zeros. The only exception is for Range-Range Rate data where Words 17-21 have the following information:

Word 17 - quality bit $\begin{cases} 0 \text{ good data} \\ 1 \text{ bad data} \end{cases}$

Word 18 - C_2

Word 19 - C_4

Word 20 - C_3

Word 21 - C_1

This tape is completely in line with MINIVAR requirements.

At the end of the run, the number of points written is output on A3.

2.1.4.2.6 Subroutines Requires

2.1.4.2.6.1 MINIT

Purpose: Read and format MINITRACK observation data for merging with the other system types.

Special Features

1. When two data times (an ℓ direction cosine and an m direction cosine) from any one station fall within 0.1 milliseconds of each other, the data will be made into one observation consisting of ℓ and m direction cosines.

2. If the station name from INPUT does not match the observation name, the data are rejected.

3. If the observation type is not 2 or 3 (see data format), the data are rejected.

2.1.4.2.6.2 RANGE:

Purpose: Read and format R-RDOT observation for merging with the other system types.

Special Features

1. The raw data are broken into 9 different measurements.

   a) X and Y antenna angles

   b) 1st range Δ t

   c) 1st range rate Δ t of doppler count

   d) 2nd range Δ t

   e) 2nd range rate Δ t of doppler count

   f) 3rd range Δ t

   g) 3rd range rate Δ t of doppler count

   h) 4th range Δ t

   i) 4th range rate Δ t of doppler count

2. The time of the measurement is adjusted for the following factors:

   a) WWV delay

   b) Recording Rate*

   c) Recording delay*

   d) Transponder delay

*See Appendix I for details.

3. For range rate Δ t of doppler count, the decimal point is assumed at the extreme left (before 1st digit). For range Δ t's, the decimal point is either at the extreme left (before 1st digit) or between first digit and second digit depending on whether C_4 (range counter frequency indicator) is either odd or even, respectively. For an odd C_4, the range counter frequency is 100 MC; for an even C_4, the range counter frequency is 10 MC.

2.1.4.2.6.3 DATGEN

<u>Purpose</u>: Break up a double precision time measurement in hours into integral hours, integral minutes beyond integral hours, and integral and fractional seconds beyond integral minutes.

2.1.4.2.6.4 EOFIX

<u>Purpose</u>: To avoid termination by EXEM because of an end of file.

<u>Program Sequence</u>

```
      CALL EOFIX (IND)
      IF (IND) 1,2,1
2     READ ---
      CALL EXEMR
      .
      .
      .
1     CALL EXEMR
```

IND = 0 no end of file
= 1 end of file encountered

CALL EXEMR - resets EXEM to normal condition

2.2 <u>Alternate 1</u>

2.2.1 Form of Data

<u>$R/\dot{R}$</u> - The $R/\dot{R}$ system data (raw) is available on 30 day tapes, the data being in sections corresponding to a pass of the satellite over a station. The data in each section must be time ordered, but the sections need not be in order.

<u>Minitrack</u> - The ℓ and m data, already edited, must be put on cards and mechanically time ordered; then placed back on tape (or use IBSYS SORT).

2.2.2 Format

See Appendices I and J for formats of the data as they are made available.

2.2.3 Data Flow

The following diagram illustrates the data flow in this mode.

2.2.4 Required Programs

2.2.4.1 SPSORT

2.2.4.1.1 Purpose

This routine does a preliminary editing of the raw R & Ṙ data (available in the format described in 2.2.1 and 2.2.2 above) and places the edited data on n tapes (n corresponding to the number of stations) for use by a later program which time orders the data from all n stations.

2.2.4.1.2 Requirements

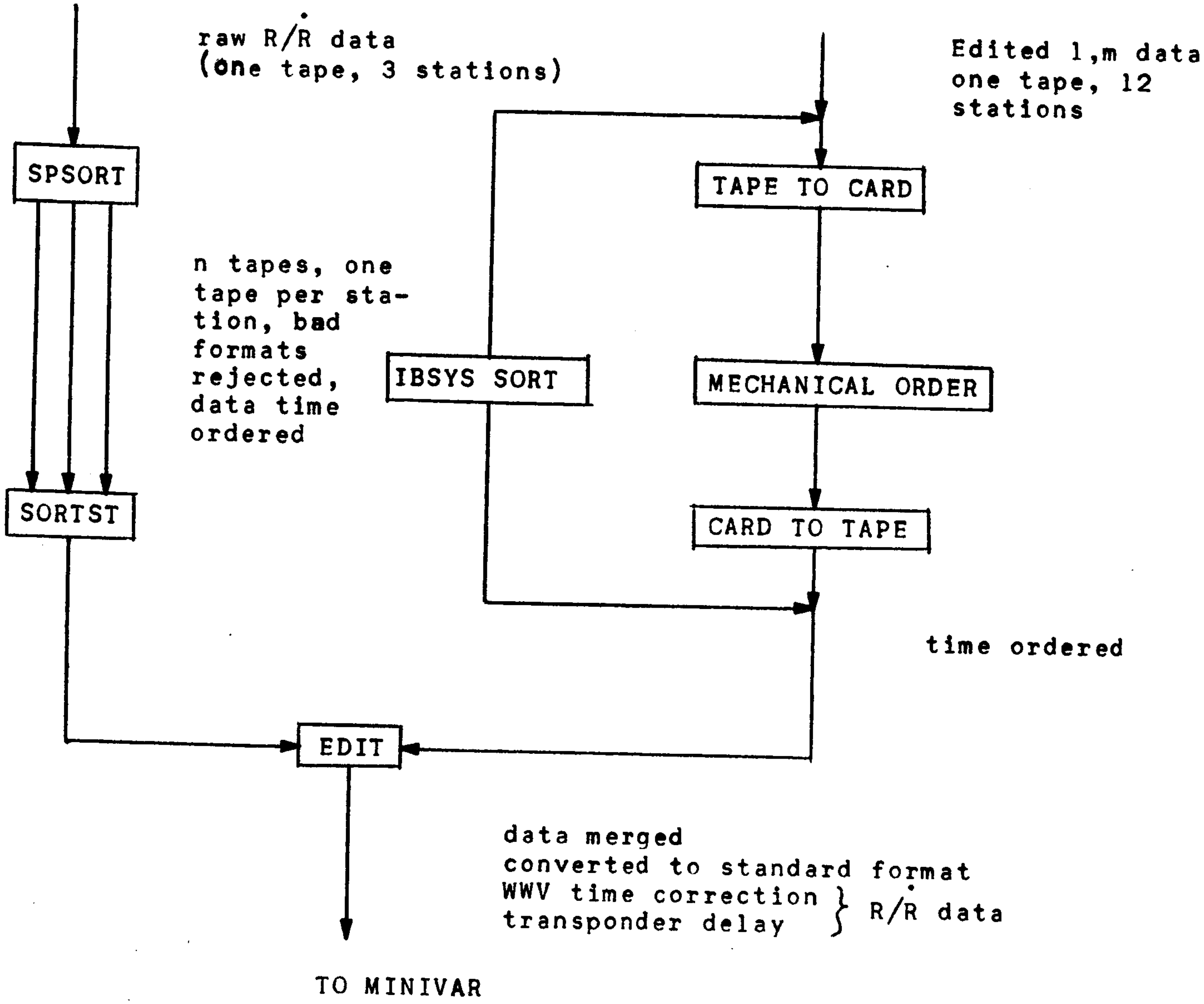
a. Successive points from any one station are in perfect time order.

b. The tape has an end of file at end of data.

c. The data have the format described in 2.1.4.1.2.

2.2.4.1.3 Method

Data points are continuously read from the input tape. If the data set is from the station being sorted, it is written out. Otherwise, the data are rejected. Other reasons for rejection of data are given below.


raw R/R data
(one tape, 3 stations)
Edited l,m data
one tape, 12
stations
SPSORT
TAPE TO CARD
n tapes, one
tape per sta-
tion, bad
formats
rejected,
data time
ordered
IBSYS SORT
MECHANICAL ORDER
SORTST
CARD TO TAPE
time ordered
EDIT
data merged
converted to standard format
WWV time correction
transponder delay
R/R data
TO MINIVAR

2.2.4.1.4 Data Rejection

Data will be rejected for the following reasons:

a. Data do not correspond to input station number

b. line identification ("." or "/") not present

c. date of data set selected for writing is less than those already written (for that station)

d. last $\dot{R}$ digit of each line is a space

e. Any non-numeric information in X,Y,R,$\dot{R}$,D,H,M,S,STA, C_1,C_2,C_3,C_4 data fields.

2.2.4.1.5 Input

CARD	VALUE	COL	PURPOSE
1	ISTA	1-5	Particular R-$\dot{R}$ station being broken out onto a separate tape
2	ISTA	1-5	Particular R-$\dot{R}$ station being broken out onto a separate tape
3	ISTA	1-5	Particular R-$\dot{R}$ station being broken out onto a separate tape

Maximum of 3 cards, hence maximum of 3 stations. If there are less than three stations, then use only a corresponding number of input cards. Data tape is placed on B5.

2.2.4.1.6 Output

The output is written on B8, B9, or B10 corresponding respectively to pass 1, 2, or 3 in the exact format as read from B5. The total number of points written is output on A3.

2.2.4.1.7 Subroutines Required

2.2.4.1.7.1 MASK - a FAP routine

See 2.1.4.1.7.1

2.2.4.1.7.2 EOFIX - A FAP routine

See 2.1.4.1.7.2.

2.2.4.2 <u>SORST</u>

2.2.4.2.1 Purpose

This routine can do the preliminary editing of the raw R & $\dot{R}$ data. However, in this mode it serves only as a vehicle for merging $n \leq 3$ separate previously edited R/$\dot{R}$ data tapes onto one in such a manner that they are time ordered.

2.2.4.2.2 Requirements

a. Each input tape has only 1 station's data

b. The data on each tape are time ordered

c. Each tape has an end of file at the end of the data

d. The data format is as prescribed in 2.1.4.1.2.

2.2.4.2.3 Method

One data set is read from each tape and the lowest date is selected for writing. A set from a station tape to which the written set corresponded is read into core to replace it. The process is then continued until an end file is encountered on each station tape.

2.2.4.2.4 Data Rejection

See 2.2.4.1.4.

2.2.4.2.5 Input

See 2.1.4.1.5

2.2.4.2.6 Output

See 2.1.4.1.6.

2.2.4.2.7 Subroutines Required

2.2.4.2.7.1 MASK

See 2.1.4.1.7.1.

2.2.4.2.7.2 EOFIX

See 2.1.4.1.7.2.

2.2.4.2 EDIT

See 2.1.4.2.

2.3 Alternate 2

2.3.1 Form of Data

$R/\dot{R}$ - in this alternate, it is assumed that the raw $R/\dot{R}$ data are available on a tape per station basis with the data time ordered.

Minitrack - the ℓ and m data, already edited, can be put on cards and mechanically time ordered; then placed back on tape (or sorted by IBSYS sort).

2.3.2 Format

See Appendices I and J for formats of the data as they are made available.

2.3.3 Data Flow

The following diagram illustrates the data flow in this mode.

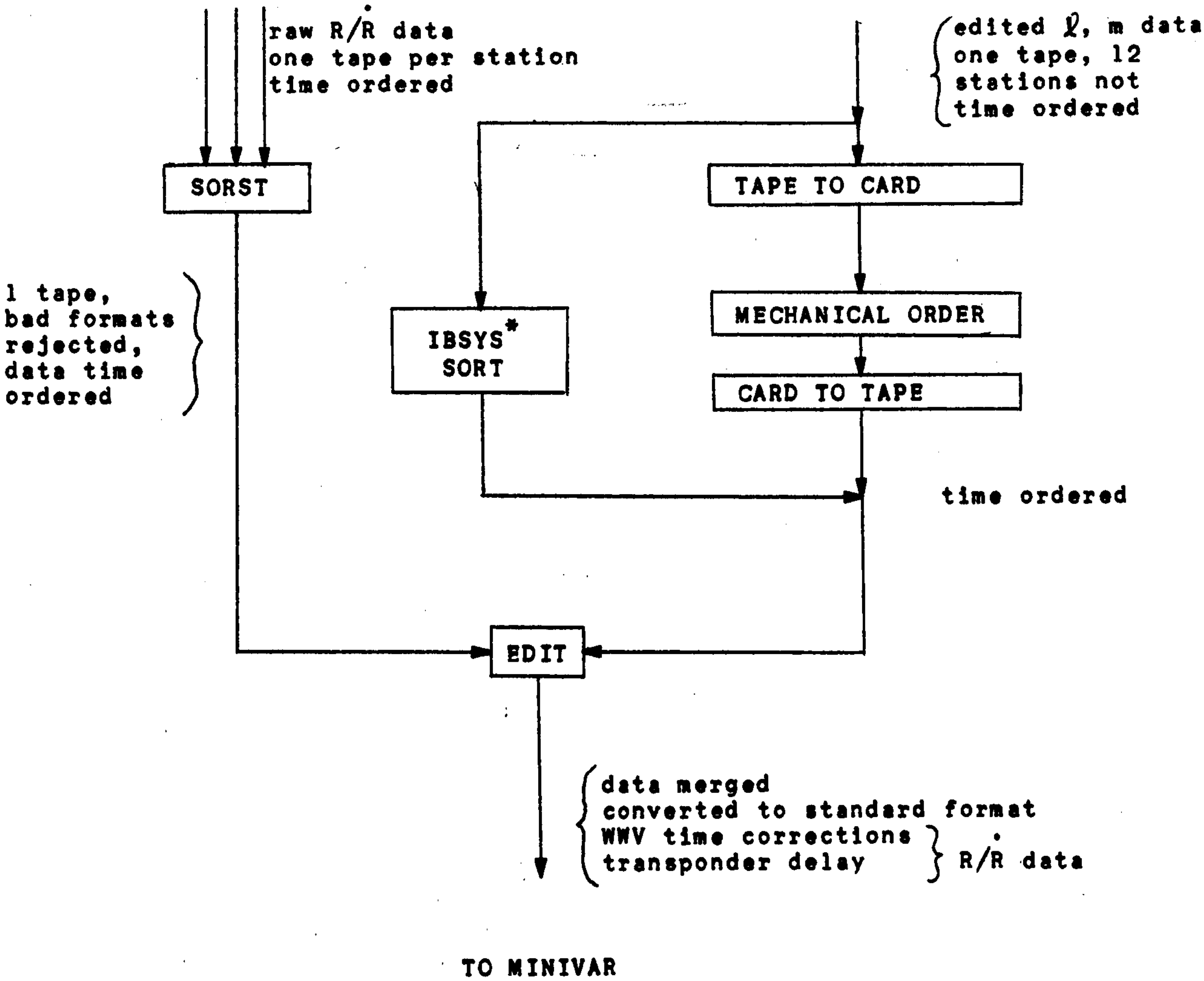
2.3.4 Required Programs

2.3.4.1 SORST

See 2.2.4.2.

2.3.4.2 EDIT

See 2.1.4.2.



* or any SORT program.

Tracking Systems

APPENDIX I

THE GODDARD RANGE AND RANGE RATE SYSTEM

1.0 System Concept and Description

The primary function of the Range and Range Rate system is to derive precise data for use in trajectory determination of the aerospace vehicles. The principal data types are topocentric range, range rate and angular information from each of 3 stations.

The prime operational objective of the system is a transportable satellite tracking system which is simple in concept, uncomplicated in operation, and dependable in performance. The design objective is a system capable of determining range with an instrument precision of ±15 meters, range rate to ±0.1 meters/second, and angular data to ±0.1 degree when used in conjunction with appropriate vehicle transponders and antennas at data rates of 8,4,2, and 1 times per second (for R and $\dot{R}$) and once per second (for angles). The system is synchronized in time to WWV with a precision of better than 1 millisecond.

For range measurements the system uses a basic sidetone ranging technique; for range rate it uses a coherent Doppler technique. Angular data are obtained from X-Y mounted antennas in order to obtain accurate and continuous data around zenith.

Two separate radio frequency channels are provided between the tracking station and the space vehicle. One channel is provided at S-band for precision tracking to reduce the propagation anomalies associated with lower frequencies. The other channel is at VHF to provide tracking when smaller transponders are required. Also, the

VHF system provides a broad beamwidth antenna pattern for easier and faster acquisition.

One mode of system operation is to extract range, range rate, and angular data from the S-band portion of the system while utilizing the VHF receiving system for the initial antenna acquisition function. In this configuration, the relatively broad beamwidth (approximately 16 degrees) of the VHF antenna will sector scan around the expected approach azimuth angle until the VHF receiver locks onto the 136 - 137 Mc Minitrack beacon transmissions from the satellite. During this antenna acquisition function, the S-band antenna (approximately 2.5 degrees beamwidth) is slaved to the VHF antenna and the S-band transmitter is radiating. After the VHF antenna acquires, the satellite transponder is unsquelched by the S-band carrier frequency and begins transmitting, since the S-band antenna is pointing toward the satellite by virtue of the slaving operation. The S-band receiver then locks onto the satellite transmissions and automatically begins modulation of the ground transmitter carrier frequency with the range tones. At the same time, the S-band antenna system automatically adopts an autotrack mode of operation. The S-band system is then independent of the VHF system and is extracting range, range rate, and angular data.

Another mode of system operation is to acquire and track at the VHF frequency. In this instance, the acquisition beacon will, in actuality, be a single-channel VHF transponder developed and supplied by NASA. The VHF transponder carrier frequency will be acquired by the VHF ground receiver while the VHF antenna is sector scanned. The antenna will automatically change to the autotrack mode, and range tones will modulate the VHF ground transmitter carrier frequency after carrier lock in the ground receiver. The VHF system will then begin extracting range and range rate data. Antenna angular readouts are not included as a part of the VHF antenna system. Angular data can be obtained during VHF operation by slaving the S-band antenna to the VHF antenna and recording the S-band antenna angular readout data. The highest ranging tone presently contemplated for use with the VHF transponder is 20 kc., due to frequency band allocation restrictions. A 100-kc range tone frequency capability is provided in the ground equipment for possible future usage.

The VHF and S-band ground system antennas are mounted on identical X-Y pedestals which are hydraulically powered. The VHF antenna system consists of a 28-foot square array of cavity backed slots. It provides 20 db of gain at the transmit frequency, 19 db of gain at the receive frequency, and it operates as a phase monopulse tracking antenna. The S-band antenna system consists of two 14-foot parabolas mounted side by side on the pedestal. One is used for transmitting with a gain of 35 db. The other parabolic antenna is used for receiving with a gain of 33 db and utilizes amplitude monopulse tracking. The use of separate transmitting and receiving S-band antennas permits a considerably lower effective receiving antenna noise temperature. The distance between the two antenna systems is about 400 feet.

Ten kilowatts of ground transmitter power are utilized for both the VHF and S-band equipments, yielding adequate signal strengths to the up-link. The power output of the transmitters is switchable to a 1-kw level for close-in tracking, if necessary to avoid transponder saturation.

Range measurements are accomplished by the use of ranging sidetones where the phase delay of these ranging sidetone frequencies is directly proportional to the two-way range between the tracking antenna and the satellite. The ranging sidetone frequencies are 500 kc, 100 kc, 20 kc, 4 kc, 800 cps, 160 cps, 32 cps, and 8 cps tones. The highest frequency is selectable between 500 kc, 100 kc and 20 kc and is used to determine the finest increment of range, and the lower frequency tones are used to remove range measurement ambiguities. The five-to-one frequency difference is used in order to maintain a maximum ability in achieving automatic ambiguity resolution of the highest sidetone frequency used.

The lowest frequency used is selectable between 160 cps, 32 cps and 8 cps and will be determined by the maximum expected range during a particular tracking mission. The range tones are phase modulated onto the ground transmitter carrier frequency and transmitted to the transponder. The transponder retransmits these tones, maintaining ranging-tone phase coherence. The ground receiver recovers the ranging-tone frequencies and decomplements. The extractor unit functions to compare the phase of the received ranging tones with the transmitted ranging-tone phase to determine range between the tracking station and the satellite.

Range rate or satellite velocity relative to the tracking station is determined by measuring the received Doppler cycles per unit of time, to determine the average velocity or range rate over that time interval. In order to determine the Doppler frequency, it is necessary to maintain frequency coherence of the ground transmitter frequency through the transponder and back to the ground receiver where it is compared in frequency against a coherent sample of the transmitter frequency.

2.0 Data Recording and Format

2.1 Format

The data recording system, at least as far as the timing involved, is quite complex. The explanation of the system will start with a description of the punch paper tape format (when printed).

First Line

XXXXXQ_1RRRRRRRR$Q_2\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}$"DDDHHMMSS$Q_3$RRRRRRRR$Q_4\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}$

Second Line

SSSSS
YYYYYQ_5RRRRRRRR$Q_6\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}$/AAATTCCCC$Q_7$RRRRRRRR$Q_4\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}\dot{R}$
TTTAA1234

X antenna position (sign and four places)

Y antenna position (sign and four places)

R Range (indicated at a Δ t)

$\dot{R}$ Range rate (indicated by a Δ t)

Q_I Quality of data-a space is good data, a(?) is faulty data; indicator precedes data.

D Days

H	Hours (Universal Time)
M	Minutes
S	Seconds (integer)
SAT	Three digit satellite identification
STA	Station identification
C_1	Ambiguity and Resolution indicator
C_2	Recording Rate and punch ID
C_3	S-band or VHF frequencies
C_4	Range counter frequencies (10 or 100 mc)

2.2 Timing and Data Rate

The format described above contains 1 X-angle, 1 Y-angle, 4 R and 4 $\dot{R}$ data points. The R and $\dot{R}$ points are equally spaced with the space being determined by the recording rate, as indicated by the character C_2.

$R/\dot{R}$ Sampling Rate	Format Printing Time
8*/sec	1 sec
4/sec	1 sec
2/sec	2 sec
1/sec	4 sec

*At the 8 per second rate, two punches are required, each punching at a 4 per second rate. Every other R and $\dot{R}$ pair are printed on alternate tapes.

The time recorded on each frame is the integer second that starts the frame, as determined by a timing system synchronized to WWV.

The X and Y axis angles are sampled 30 microseconds following the start-of-the-second given by the time printout. Thus, the X and Y position sampling and printout rates will be once per second, once per two seconds or once per four seconds, respectively.

The start of the range measurement occurs 25, 30 or 31 microseconds following the reference pulses, depending on whether the highest range tone used is 20 kc, 100 kc, or 500 kc, respectively. Table I indicates the timing sequence, t_o is the recorded time, D is the small delay described above.

The start of the range rate measurement always begins with the second zero crossing of the Doppler (+ bias) sinusoid following the reference pulse. Since this sinusoid is not coherently related to the reference pulse, range rate measurements start anywhere between the limits $1/(f_{D+b})$ and $2/(f_{D+b})$, where (f_{D+b}) = Doppler plus bias frequency; b = 500 kc at S-band; b = 30 kc at VHF. Table I also applies to the range rate system, except D lies between the limits just described.

2.3 C-Character Definitions

The $C_1 \longrightarrow C_4$ characters contain information necessary to the program. Their use is in the following areas: ambiguity resolution, accuracy resolution in range and range rate, and estimating the error in the measurement. Tables II, III, IV and V indicate the significance of each character.

Recording Rate	1st range	2nd range	3rd range	4th range
8/sec Punch A1	to + D sec	to + $\frac{1}{4}$ + D	to + $\frac{1}{2}$ + D	to + 3/4 + D
8/sec Punch A2	to + 1/8 + D	to + 3/8 + D	to + 5/8 + D	to + 7/8 + D
4/sec	to + D	to + $\frac{1}{4}$ + D	to + $\frac{1}{2}$ + D	to + 3/4 + D
2/sec	to + D	to + $\frac{1}{2}$ + D	to + 1 + D	to + 1 $\frac{1}{2}$ + D
1/sec	to + D	to + 1 + D	to + 2 + D	to + 3 + D

TABLE I-I: INTERNAL TIMING OF DATA FRAME

C_1 DIGIT	AMBIGUITY (f_L)			RESOLUTION (f)		
	160	32	8	20 KC	100 KC	500 KC
1			X	X		
2			X		X	
3			X			X
4		X		X		
5		X			X	
6		X				X
7	X			X		
8	X				X	
9	X					X

C_2 DIGIT	PUNCH #		RECORDING RATE (C/S)			
	1	2	1	2	4	8
0	X		X			
1	X			X		
2	X				X	
3	X					X
4		X	X			
5		X		X		
6		X			X	
7		X				X

TABLES I-II AND III: C_1 AND C_2 SYMBOL DEFINITIONS

C_3 DIGIT	BIAS FREQUENCY	CARRIER FREQUENCY	
		Up Frequency	Down Frequency
0	500 KC	2270.1328 mc	1705.0 mc
1	500 KC	2270.9328 mc	1705.0 mc
2	500 KC	2271.9328 mc	1705.0 mc
3	30 KC	148.260 mc	136 mc

C_4 DIGIT	X AXIS	Y AXIS	RANGE COUNTER
0	slave	slave	10 mc
1	slave	slave	100 mc
2	slave	auto	10 mc
3	slave	auto	100 mc
4	auto	slave	10 mc
5	auto	slave	100 mc
6	auto	auto	10 mc
7	auto	auto	100 mc
8	neither	neither	10 mc
9	neither	neither	100 mc

TABLES I-IV AND V: C_3 AND C_4 SYMBOL DEFINITIONS

a. Ambiguity Resolution

The C1 digit indicates the lowest frequency employed in the phase instrumented ranging system. This information is employed in the ambiguity resolution system described in Appendix D.

b. Accuracy Resolution

The combined information in C_2 and C_3 is used to resolve $\dot{R}$. The range rate (one way) is given by

$$\dot{\rho} = \frac{c}{2f_\mu}\left(K - \frac{N}{\Delta t_p'}\right)$$

where

c is the velocity of light
f_μ is the up frequency (defined by C3)
K is the bias frequency (defined by C_3)
N is the cycle count (defined below)
$\Delta t_p'$ is the $\dot{R}$ reading in the data format

N is given by

	C_3	
C_2	0, 1, 2	3
0, 4	229262	14328
1, 5	131007	8187
2, 6	65503	4093
3, 7	32751	2046

c. Accuracy Estimation

The C4 digit is an indication of parts of the system which are employed in making a measurement. These, in turn, provide a basis for estimating the accuracy of the instrumentation in X, Y and range.

3.0 Station Parameters

Sites now in use or planned for the system and pertinent information are as follows:

Rosman, N. C.

Station number	26
Status of site	permanent
Geodetic net	Vanguard
WWV correction	3.6 milliseconds

VHF antenna

geodetic parameters

Latitude	35°11' 41.100"
Longitude	277° 7' 26.253"
Height	2890 feet

data precision (instrument only)

$\sigma_\rho = 75$ m $\qquad \sigma_{\dot\rho} = .75$ m/s

$\sigma_x = 2.5^\circ \qquad \sigma_y = 2.5^\circ$

UHF antenna

geodetic parameters

Latitude

Longitude

Height above geoid

data precision (instrument only)

Not presently used

Scottsdale, Arizona

Station number	64
Status of site	Mobile
Geodetic net	Vanguard
WWV correction	11.45 ms

VHF antenna

geodetic parameters

Latitude $33^\circ\ 27'\ 42.740''$

Longitude $248^\circ\ 5'\ 52.720''$

Height 1000 feet

data precision (instrument only)

$\sigma_\rho = 75$ m $\quad \sigma_{\dot\rho} = .75$ m/s

$\sigma_x = 2.5^\circ \quad \sigma_y = 2.5^\circ$

UHF antenna

geodetic parameters

Latitude

Longitude

Height above geoid

data precision (instrument only)

Not presently used

<u>Australia</u>

Parameters not yet designated.

APPENDIX J

THE MINITRACK SYSTEM

1.0 System Concept and Description

The primary function of the Minitrack system is to provide very precise angular data for the determination of trajectories of aerospace vehicles. The data types are the direction cosines of the angles between the station-to-vehicle vector and the station's geodetic east and geodetic north vectors. After proper data editing, time correction (for WWV delay), and calibration, a precision of .00025 (in direction cosine) is achieved in both channels. Bias errors due to propagation are not included in this error estimate; at the frequency employed these errors can be severe. For this reason, the antenna system has not been engineered to receive signals below 60° from the zenith, thus very high refraction errors at low elevation angles are not encountered. Figure J-1 shows the relationship between the standard deviation in direction cosine to its equivalent angular error for 3 different values of the standard deviation in direction cosine.

The system employs a 136 mc CW oscillator in the vehicle. The ground system is completely passive; no ground-to-vehicle transmission is needed. The ground antenna and receiver configuration is quite complex since the separate east and north channels each contain suffient components to resolve ambiguities in the CW phase measuring system employed.

Figure J-2 illustrates the relationship of the phase difference between the signal, as received at each antenna, and the angle, β. When the baseline length, B, is greater than the wavelength, λ, ambiguities result since $\Delta\lambda$ can change through more than one wave length of the carrier frequency, f_o, as β varies over the range $\pm \pi/2$. These ambiguities are resolved by using additional antennas whose baseline lengths, B, are much shorter than the baseline used for the highest precision measurement.

The required quantity, ℓ, is related to the angle and the actual measurement by

$$\ell = \cos\beta = \frac{\Delta\lambda}{B}$$

The Minitrack stations were originally designed for tracking of vehicles with inclinations below 60°. A fence of stations, running in a roughly N-S direction along the east coast of North America and the west coast of South America was established to provide at least one tracking interval per orbit.

Using this philosophy, the beamwidth of the antennas was designed to be roughly 90° in the NS direction and 12° in the EW direction. Thus, the data in the ℓ-direction cosine, taken at angles nearer 90°, provided more accurate information, on the average, than the m-direction cosine information.

With the advent of satellites in more nearly polar orbits, some of the stations were outfitted with antennas which have their antenna patterns rotated 90°, i.e., the 6° beamwidth pattern is in the NS

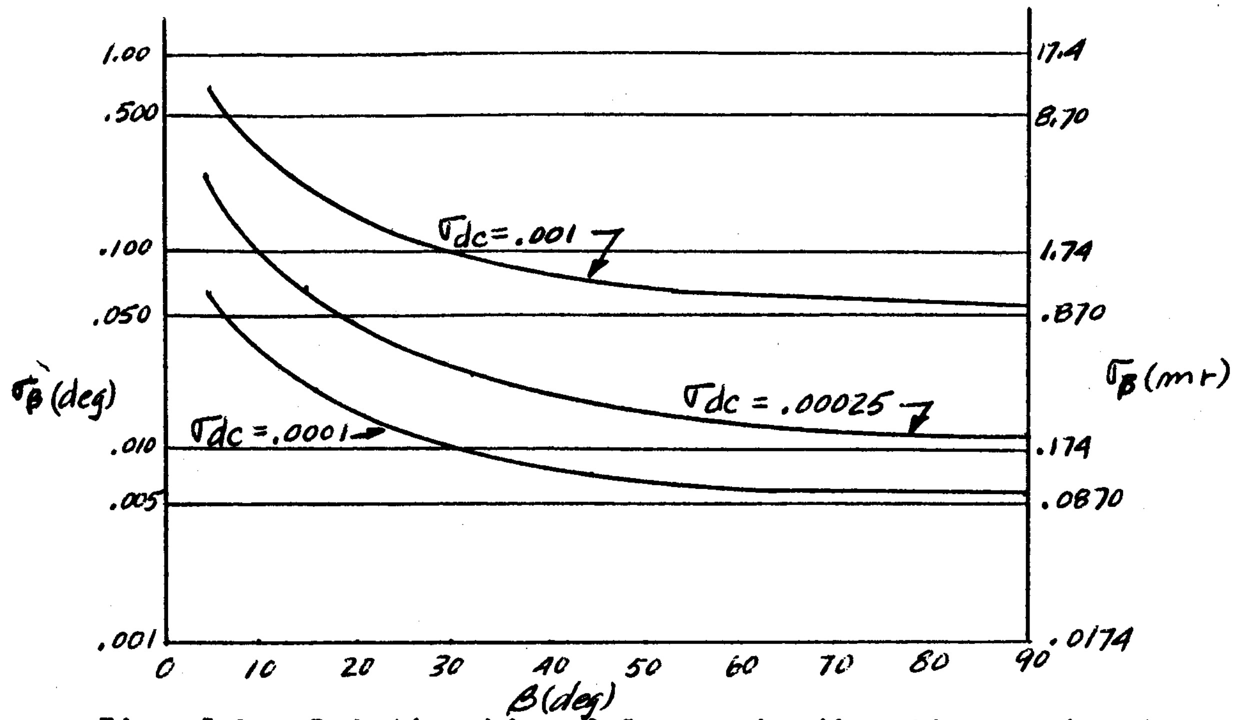


Fig. J-1: Relationship of error in direction cosine to error in angle

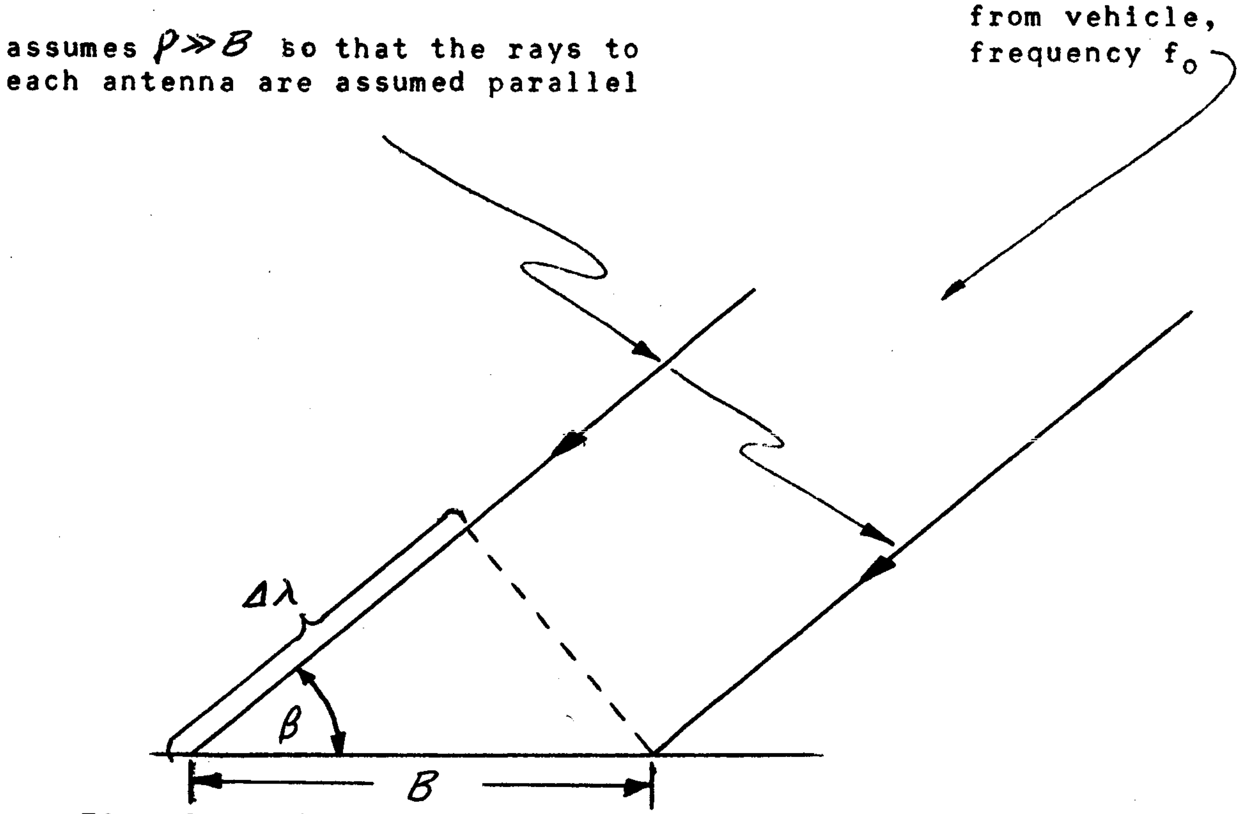


Fig. J-2: Geometry of the Minitrack measuring system direction, the 90° beamwidth pattern is in the EW direction. This information is included in the data; it does not alter the accuracy or data format of the system.

Thus, a high accuracy system is achieved for both polar and equatorially oriented satellites without requiring complex orbiting equipment. The data give the direction of the station-to-vehicle vector, but not the magnitude.

This information provides reasonably high precision orbit determination. When combined with the high accuracy Goddard $R/\dot{R}$ system, a very effective tracking system results.

2.0 Data Editing and Format

2.1 Raw Data Editing

The raw data from each of the tracking stations are received at Goddard. Considerable data concerning calibration information are included in the raw data format. Prior to its use in a trajectory determination program, the information is processed so that calibration and ambiguity resolution problems are removed and obviously incorrect data points are deleted. In addition, blocks of 5 data points are combined in a best least-squares fit to a second-order polynomial. The output information from the program is the direction cosine value from this least-squares fit at the mid-point, in time, of the data. The data, in their proper format, are used in the orbit determination program.

2.2 Data Format

The raw Minitrack data, after processing, are output on magnetic tape or cards in the format to be described below:

Col.	1	blank
	2,3	year of launch
	4,5	launch number
	6	object number
	7	blank
	8-13	station designation (e.g., SNTAG6)
	14	blank
	15,16	year of data
	17,18	month of data
	19,20	day of data
	21	blank
	22,23	hour of data
	24,25	minute of data
	26	blank
	27-31	second of data (decimal assumed between columns 28 & 29)
	32	blank
	33-43	data grading (not recommended for further use)
	44-59	blank
	60	sign (assumed plus if blank)
	61-67	direction cosine value (decimal assumed between columns 61 & 62)
	68-70	blank
	71	antenna indicator, E for equatorial mode, P for polar
	72	type indicator, 2 = ℓ - direction cosine 3 = m - direction cosine
	73-80	blank

3.0 Station Parameters

Sites now in use are:

Fairbanks, Alaska

station designation	COLEG6
datum	Vanguard
WWV correction	27.4 ms
geodetic parameters	
latitude	64° 52' 18.591"
longitude	212° 9' 47.387"
height	527 feet
data precision	

$\sigma_\ell = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

Blossom Point, Maryland

Station designation	BPOIN6
datum	Vanguard
WWV correction	0.5 ms
geodetic parameters	
latitude	38° 25' 49.718"
longitude	282° 54' 48.170"
height	15 feet
data precision	

$\sigma_\ell = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

Fort Meyers, Florida

Station designation	FTMYR6
datum	Vanguard
WWV correction	5.8 ms

geodetic parameters

latitude 26° 32' 53.516"

longitude 278° 8' 3.887"

height 11.56 feet

data precision

$\sigma_l = .00025 \quad \sigma_m = .00025 \quad \rho_{lm} = 0$

East Ground Forks, Minnesota

station designation GFORK6

datum Vanguard

WWV correction 7.2 ms

geodetic parameters

latitude 48° 1' 20.668"

longitude 262° 59' 21.556"

height 823.0 feet

data precision

$\sigma_l = .00025 \quad \sigma_m = .00025 \quad \rho_{lm} = 0$

Hartebeeshoek, South Africa

station designation JOBUR6

geodetic net E30Eur.

WWV correction 45.7 ms

geodetic parameters

latitude -25° 52' 58.862

longitude 27° 42' 27.931

height 4988 feet

data precision

$\sigma_l = .00025, \quad \sigma_m = .00025 \quad \rho_{lm} = 0$

Lima, Peru

station designation LIMAP6

datum Vanguard

WWV correction 20.2 ms

geodetic parameters

latitude $-11^\circ\ 46'\ 36.492''$

longitude $282^\circ\ 50'\ 58.184''$

height 161 feet

data precision

$\sigma_l = .00025 \qquad \sigma_m = .00025 \qquad \rho_{lm} = 0$

Goldstone Lake, California

station designation MOJAV6

datum Vanguard

WWV correction 12.6 ms

geodetic parameters

latitude $35^\circ\ 19'\ 48.525''$

longitude $243^\circ\ 6'\ 2.776''$

height 3044 feet

data precision

$\sigma_l = .00025 \qquad \sigma_m = .00025 \qquad \rho_{lm} = 0$

Island Lagoon, Australia

station designation OOMER6

datum Australian

WWV correction 59.1 ms

geodetic parameters

latitude -31° 23' 30.638"

longitude 136° 52' 19.634"

height 436 feet

data precision

$\sigma_\ell = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

<u>Quito, Equador</u>

station designation QUITO6

datum Vanguard

WWV correction 16.1 ms

geodetic parameters

latitude -0° 37' 21.751"

longitude 281° 25' 14.770"

height 11,703 feet

data precision

$\sigma_\ell = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

<u>Santiago, Chile</u>

station designation SNTAG6

datum Vanguard

WWV correction 28.7 ms

geodetic parameters

latitude -33° 8' 58.106"

longitude 289° 19' 51.283"

height 2280 feet

data precision

$\sigma_\ell = .00025,$ $\sigma_m = .00025,$ $\rho_{\ell m} = 0$

Winkfield, England

station designation	WNKFL6
datum	E30Eur.
WWV correction	20.8 ms
geodetic parameters	
latitude	51° 26' 44.122"
longitude	359° 18' 14.615"
height	215 feet
data precision	

$\sigma_\ell = .00025$, $\sigma_m = .00025$ $\rho_{\ell m} = 0$

St. Johns, Newfoundland

station designation	NEWFL6
datum	Vanguard
WWV correction	8.1 ms
geodetic parameters	
latitude	47° 44' 29.049"
longitude	307° 16' 43.240"
height	208 feet
data precision	

$\sigma_\ell = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

APPENDIX K

CORRECTIONS FOR GEODETIC NET ERROR

Kaula (Ref. 18) has recommended that the principal geodetic nets now in use as the basis for station location information are shifted with respect to each other. These errors, given in a coordinate system with u toward Greenwich, w toward the pole and v completing the system, are given below.

For stations connected to the principal geodetic control system of the Western Hemisphere:

	Meters		
	Δu	Δv	Δw
N. American Datum	-23	+142	+196
S. American Datum	-303	+ 98	-344
SAO SP 59	- 6	+212	- 96
Vanguard	- 22	+147	+ 9
σ	26	22	22

For stations connected to the Europe-Africa-Siberia-India geodetic system:

	Meters		
	Δu	Δv	Δw
European Datum	- 57	- 37	- 96
Indian	+200	+782	+271
Arc	-109	- 70	-289
SAO SP 59	-150	- 2	- 33
σ	23	29	23

For stations connected to the Japan-Korea-Manchuria geodetic system:

	Meters		
	Δu	Δv	Δw
Tokyo	- 89	+551	+710
SAO SP 59	+ 37	-268 ,	- 82
σ	40	53	40

For stations connected to the Australian system:

	Meters		
	Δu	Δv	Δw
Sydney	+368	+173	-290
SAO SP 59	+192	-106	+ 44
σ	75	90	35

For stations connected to the Argentine system:

	Meters		
	Δu	Δv	Δw
SAO SP 59	-107	+173	+ 34
σ	180	160	16

It should be emphasized that for stations connected to the same geodetic control system, the errors between stations should be less than ±20 meters.

24,25	minute of data
26	blank
27-31	second of data (decimal assumed between columns 28 and 29)
32	blank
33-34	data grading (not recommended for further use)
44-59	blank
60	sign (assumed plus if blank)
61-67	direction cosine value (decimal assumed between columns 61 and 62)
68-70	blank
71	antenna indicator, E for equatorial moce, P for polar mode
72	type indicator, 2 = ℓ - direction cosine 3 = m - direction cosine
73-80	blank

KAULA'S ELLIPSOID

STATION PARAMETERS

Sites now in use are:

Fairbanks, Alaska

Station designation COLEG6

Station number (1)

datum Kaula's Ellipsoid

WWV correction 27.4

geodetic parameters

latitude $64^{\circ}52'18''.119$

longitude $-147^{\circ}50'22''.953$

height 563.0 feet

data precision

$\sigma_{\ell} = .00025 \quad \sigma_m = .00025 \quad \rho_{\ell m} = 0$

Blossom Point, Maryland

Station designation BPOIN6

Station number (2)

datum Kaula's Ellipsoid

WWV correction 0.5 ms

geodetic parameters

latitude $38^{\circ}25'49''.972$

longitude $-77^{\circ}55'11''.449$

height -122.0 feet

data precision

$\sigma_{\ell} = .00025 \quad \sigma_m = .00025 \quad \rho_{\ell m} = 0$

Fort Meyers, Florida

Station designation FTMYR6

Station number (3)

datum Kaula's Ellipsoid

WWV correction 5.8 ms

geodetic parameters

latitude $26^{\circ}32'53''.500$

longitude $-81^{\circ}51'56''.148$

height -128.0 feet

data precision

$\sigma_{\ell} = .00025 \quad \sigma_m = .00025 \quad \rho_{\ell m} = 0$

East Ground Forks, Minnesota

Station designation GFORK6

Station number (4)

datum Kaula's Ellipsoid

WWV correction 7.2 ms

geodetic parameters

latitude $48^{\circ}1'21''.411$

longitude $-79^{\circ}0'38''.134$

height 693.0

data precision

$\sigma_{\ell} = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

Hartebeeshoek, South Africa

Station designation JOBUR6

Station number (5)

datum Kaula's Ellipsoid

WWV correction 45.7 ms

geodetic parameters

latitude $-25^{\circ}53'0''.225$

longitude $27^{\circ}42'27''.706$

height 5599.0 feet

data precision

$\sigma_{\ell} = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

Lima, Peru

Station designation LIMAP6

Station number (6)

datum Kaula's Ellipsoid

WWV correction 20.2 ms

geodetic parameters

latitude $-11^{\circ}46'35''.972$

longitude $-77^{\circ}9'1''.447$

height 11.0 feet

data precision

σ_m = .00025 σ_ℓ == .00025 $\rho_{\ell m}$ = 0

Goldstone Lake, California

Station designation MOJAV6

Station number (7)

datum Kaula's Ellipsoid

WWV correction 12.6 ms

geodetic parameters

latitude $35^{\circ}19'48''.163$

longitude $-116^{\circ}54'0''.6340$

height 2978.0 feet

data precision

σ_ℓ = .00025 σ_m = .00025 $\rho_{\ell m}$ = 0

St. Johns, Newfoundland

Station designation NEWFL6

Station number (8)

datum Kaula's Ellipsoid

WWV correction 8.1 ms

geodetic parameters

latitude $47^{\circ}44'29''.338$

longitude $-52^{\circ}43'13''.326$

height 118.0 feet

data precision

$\sigma_{\ell} = .00025 \qquad \sigma_m = .00025 \qquad \rho_{\ell m} = 0$

Island Lagoon, Australia

Station designation OOMER6

Number designation (9)

datum Kaula's Ellipsoid

WWV correction 59.1 ms

geodetic parameters

latitude $-31^{\circ}23'31''.059$

longitude $136^{\circ}52'5''.332$

height 644.0 feet

data precision

$\sigma_{\ell} = .00025 \qquad \sigma_m = .00025 \qquad \rho_{\ell m} = 0$

Quita, Ecuador

Station designation QUITO6

Station number (10)

datum Kaula's Ellipsoid

WWV correction 16.1 ms

geodetic parameters

latitude $-0^{\circ}36'37''.943$

longitude $-78^{\circ}34'44''.986$

height 11,560.0 feet

data precision

$\sigma_{\ell} = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

Santiago, Chile

Station designation SNTAG6

Station number (11)

datum Kaula's Ellipsoid

WWV correction 28.7 ms

geodetic parameters

latitude $-33^{\circ}8'57''.665$

longitude $-70^{\circ}40'7''.641$

height 2115. feet

data precision

$\sigma_{\ell} = .00025$ $\sigma_m = .00025$ $\rho_{\ell m} = 0$

Winkfield, England

Station designation WNKFL6

Station number (12)

datum Kaula's Ellipsoid

WWV correction 20.8 ms

geodetic parameters

latitude $51^{\circ}26'40''.639$

longitude $-0^{\circ}41'47''.337$

height 396. feet

data precision

$\sigma_{\ell} = .00025 \qquad \sigma_m = .00025 \qquad \rho_{\ell m} = 0$

Rosman, N. Carolina

Station designation ROSMAN

Station number (13)

datum Kaula's Ellipsoid

WWV correction 3.6 ms

geodetic parameters

latitude $35^{\circ}0'48''.628$

longitude $-82^{\circ}52'33''.752$

height 2725. feet

data precision

$\sigma_{\ell} = .00025 \qquad \sigma_m = .00025 \qquad \rho_{\ell m} = 0$

Scottsdale, Arizona

Station designation	SCOTTSDALE
Station number (14)	
datum	Kaula's ellipsoid
WWV correction	11.45 ms
geodetic parameters	
latitude	$33^{\circ}27'43''.071$
longitude	$-111^{\circ}54'10''.158$
height	1160.0 feet

data precision

$\sigma_{\ell} = .00025 \qquad \sigma_m = .00025 \qquad \rho_{\ell m} = 0$

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